

# The Good Shepherd Primary Catholic Voluntary Academy



## Mathematics Calculation Policy

### Mission Statement

*Our mission is to develop our children with active and creative minds,  
a sense of understanding and compassion for others and  
the courage to act on their Catholic beliefs.*

*In our school community, we celebrate our faith and we work together to achieve  
our personal potential by trying to live like Jesus and become the person that he  
wants us to be.*

Ratified On:	September 2020
Review Date:	September 2021
Chair of Governor's signature:	Mrs R Burke
Headteacher's signature:	Mrs M.H.B.Williams

# Mathematics Calculation Policy

## Concrete – Pictorial – Abstract

This policy has been adapted through a combination of the White Rose Calculation Policy, the White Rose small steps and our own pre-existing visual policy. The policy is set out in order to achieve the following aims:

- Ensure a consistent approach to the teaching of mental and written methods across the school, in line with our progression framework and subject intent.
- Provide all teaching staff with clear exemplification of how the CPA approach can be incorporated into lessons.

The images used in the policy are there to show examples of concrete and pictorial methods that could be used, these are not exclusive. Therefore this is a working document. The abstract methods are the 'end goal' and should be taught in-line with this policy.

## The Key Principals of the CPA Approach

The Concrete Pictorial Abstract (CPA) approach is a system of learning that uses physical and visual aids to build a child's understanding of abstract topics.

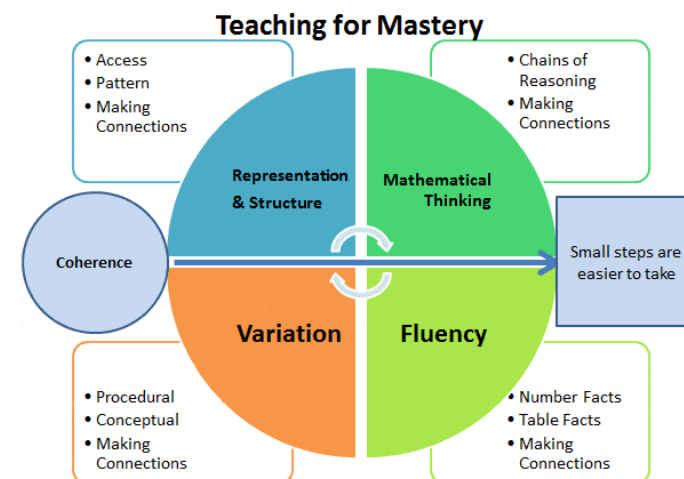
Pupils are introduced to a new mathematical concept through the use of **concrete** resources (e.g. fruit, Dienes blocks etc). When they are comfortable solving problems with physical aids, they are given problems with pictures – usually **pictorial representations** of the concrete objects they were using. Then they are asked to solve problems where they only have the **abstract** i.e. numbers or other symbols. Building these steps across a lesson can help pupils better understand the relationship between numbers and the real world, and therefore helps secure their understanding of the mathematical concept they are learning.

As part of the CPA approach, new concepts are introduced through the use of physical objects or practical equipment. These can be physically handled, enabling children to explore different mathematical concepts. These are sometimes referred to as maths manipulatives and can include ordinary household items such as straws or dice, or specific mathematical resources such as dienes or Numicon. The abstract nature of maths can be confusing for children, but through the use of concrete materials they are able to 'see' and make sense of what is actually happening. All children, regardless of ability, benefit from the use of practical resources in ensuring understanding goes beyond the learning of a procedure. They can promote reasoning and discussions, and support children in explaining a concept to others.

Once children are confident with a concept using concrete resources, they progress to drawing pictorial representations or quick sketches of the objects. By doing this, they are no longer manipulating the physical resources, but still benefit from the visual support the resources provide. This is an important step as it allows children to make the link between the concrete and the abstract.

Once children have a secure understanding of the concept through the use of concrete resources and visual images, they are then able to move on to the abstract stage. Here, children are using abstract symbols to model problems – usually numerals. To be able to access this stage effectively, children need access to the previous two stages alongside it. Some children may need to spend more time on the previous two stages, whereas others may move onto the abstract phase much sooner.

The CPA does not need to be taught in separate steps but, wherever possible, each stage should be taught simultaneously. When concrete resources, pictorial representations and abstract recordings are all used within the same activity, it ensures pupils are able to make strong links between each stage.



## **Addition and Subtraction**

### **Key Stage One**

Children first learn to connect addition and subtraction with counting, but they soon develop two very important skills: an understanding of parts and wholes, and an understanding of unitising 10s, to develop efficient and effective calculation strategies based on known number bonds and an increasing awareness of place value. Addition and subtraction are taught in a way that is interlinked to highlight the link between the two operations.

A key idea is that children will select methods and approaches based on their number sense. For example, in Year 1, when faced with  $15 - 3$  and  $15 - 13$ , they will adapt their ways of approaching the calculation appropriately. The teaching should always emphasise the importance of mathematical thinking to ensure accuracy and flexibility of approach, and the importance of using known number facts to harness their recall of bonds within 20 to support both addition and subtraction methods.

### **Lower Key Stage Two**

In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply.

In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.

### **Upper Key Stage Two**

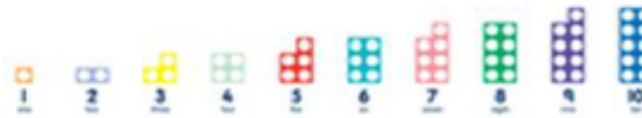
Children build on their column methods to add and subtract numbers with up to seven digits, and they adapt the methods to calculate efficiently and effectively with decimals, ensuring understanding of place value at every stage.

Children compare and contrast methods, and they select mental methods or jottings where appropriate and where these are more likely to be efficient or accurate when compared with formal column methods. Representations such as bar models are used to represent the calculations required to solve problems and may indicate where efficient methods can be chosen.

# Addition – EYFS

Numicon shapes are introduced straight away and be used to :

- order and compare number
- identify 1 more/ 1 less than
- combine pieces to add
- find number bonds



Children can record this by printing or drawing around Numicon pieces, using scales to weigh them, taking digital photographs of Numicon and placing them alongside abstract sentences.

Children begin to combine groups of objects using concrete apparatus. Children should be introduced to the part-part whole model alongside counting everyday objects.



Children make a record in pictures or words or symbols of addition activities already carried out practically. Solve simple problems using fingers.



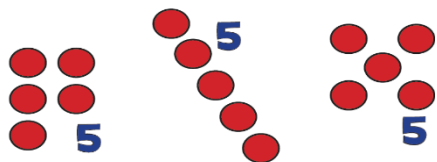
Number tracks can be introduced to count up on and to find one more and one less than. E.g. What is one more than 13? Use number lines and bead strings to start with the largest number.



Children should begin to learn how to count numbers. They should have opportunities to subitise numbers (up to 5), ordinal numbers, one to one correspondence and re-arranging objects to count them effectively.

## C1b: At a Glance

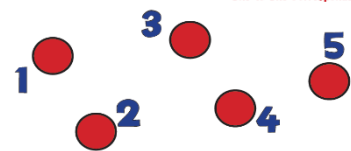
Subitising



See at a glance how many are in small collections and attach correct number names to such collections.

## C2a: Number Match

One to One Correspondence






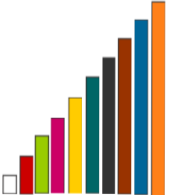

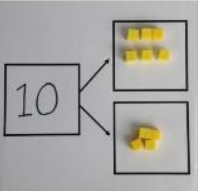

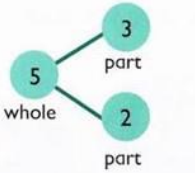
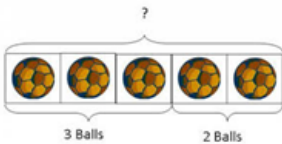



Each object to be counted must be touched or 'included' exactly once as the numbers are said.

## C4: Arranging

Sets of 5



# Addition – Year 1

Objective/strategy	Concrete	Pictorial	Abstract
<p>Finding one more.</p> <p>Autumn 1</p>	<p>Counting on and back using familiar objects and resources.</p>  <p>One more</p> 	<p>Introduce bar models to represent quantities.</p>  	<p>3 <math>\xrightarrow{\text{one more}}</math> <input type="text"/></p> <p>six <math>\xrightarrow{\text{one more}}</math> <input type="text"/></p> <p>Introduce the addition symbol (+) and equals (=) to create number sentences e.g.</p> <p><math>5 + 1 =</math></p> <p><math>7 + 1 =</math></p> <p>Missing digits e.g.</p> <p><math>5 = \square + 1</math></p>
<p>Combining two parts to make a whole: part-whole model.</p> <p>Autumn 2 Spring 1</p>	  <p>Use part, part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p> 	   <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p><b>A1a: Largest Number 1st</b></p>  <p><b>5 + 3 = 8</b></p> <p>Use part, part whole model to help children move onto the abstract:</p>  <p><math>3 + 5 = 8</math></p> <p><math>10 = 6 + 4</math></p> <p>Include missing number questions to support varied fluency e.g.</p> <p><math>3 + ? = 8</math></p>

Number bonds within 10.

Autumn 2

Break apart a group and put back together to find and form number bonds.

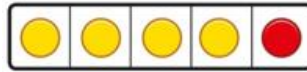


$$3 + 4 = 7$$

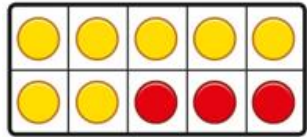


$$6 = 2 + 4$$

Use five and ten frames to represent key number bonds.

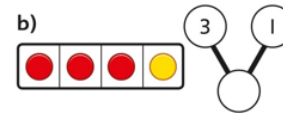
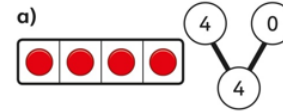


$$5 = 4 + 1$$



$$10 = 7 + 3$$

Use a part-whole model alongside other representations to find number bonds. Make sure to include examples where one of the parts is zero.



$$4 + 0 = 4$$
$$3 + 1 = 4$$

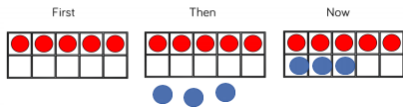
Starting at the bigger number and counting on.

Autumn 2  
Spring 2

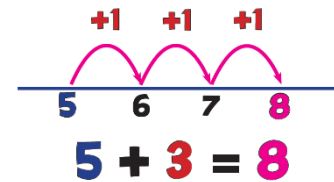


Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

Use ten frames to complete the number story.



### A2: Counting On



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

$$5 + 12 = 17$$

Place the larger number in your head and count on the smaller number to find your answer.

Regrouping to make 10.

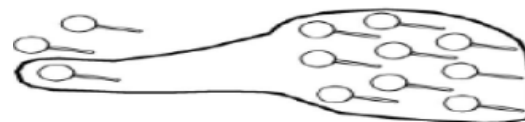
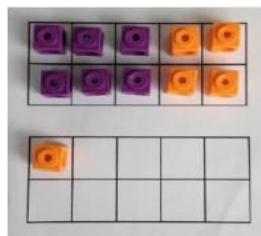
Autumn 2  
Spring 2

Start with the bigger number and use the smaller number to make 10.

Use ten frames.



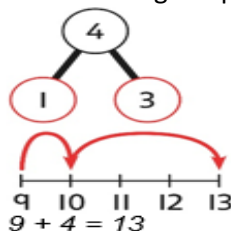
e.g.  
 $6 + 5 = 11$



$$3 + 9 =$$

Begin to use pictures and images.

Use the number line or partition the smaller number using the part part whole model.


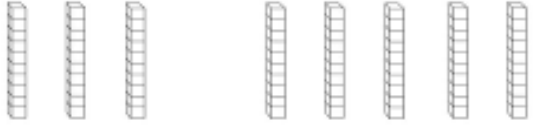
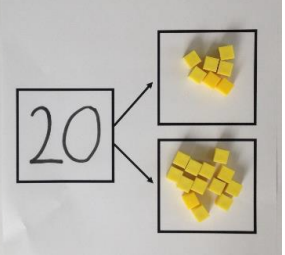
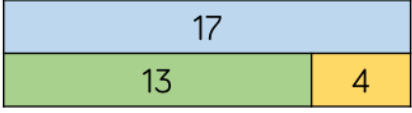

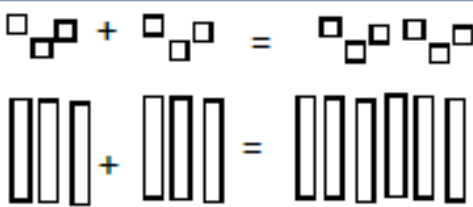
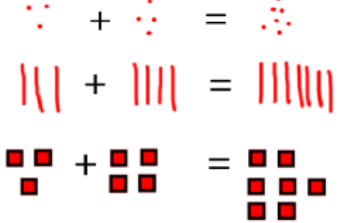


$$7 + 4 = 11$$

If I am at seven, how many more do I need to make 10. How many more do I add on now?

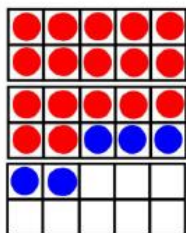


# Addition – Year 2

Objective/strategy y	Concrete	Pictorial	Abstract
Adding multiples of 10.  Autumn 2	$50 = 30 + 20$  Model using dienes and bead strings.	 $3 \text{ tens} + 5 \text{ tens} = \text{---} \text{ tens}$ $30 + 50 = \text{---}$ Use representations from base ten.	$20 + 30 = 50$  $70 = 50 + 20$  $40 + \square = 60$
Part-part whole model (making 20).  Autumn 2	 <p>Explore different ways of making numbers up to 20.</p>	Introduce bar models.   $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	Explore commutativity of addition by swapping the addends to build a fact family. Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations.  $\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using related facts.  Autumn 2	 Use dienes to represent numbers.	 Children draw the concrete representations.	$3 + 4 = 7$  So...  $30 + 40 = 70$

Add 2-digits to 1-digit numbers.

Autumn 2



Use tens frames to bridge 10.

$$17 + 5 = 22$$

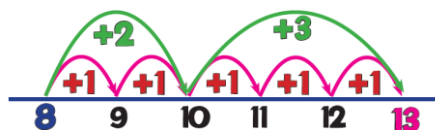
Use known bonds within 10.  
41 is 4 tens and 1 one.



41 add 6 ones is 4 tens and 7 ones.

### A2a: Counting On

<sup>1</sup> Bridging 10



$$8 + 5 = 13$$

### A2b: Counting On

<sup>2</sup> Bridging 10s Number



$$57 + 6 = 63$$

Continue to use pictorial methods if necessary.

$$8 + 5 = 13$$

$$57 + 6 = 63$$

Continue to use related facts where appropriate.

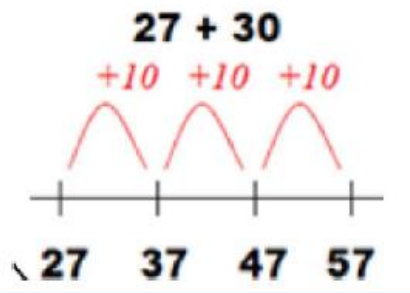
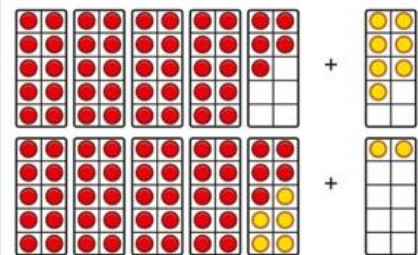
Add two 2-digit numbers (using number lines, partitioning and multiples of 10).

Autumn 2



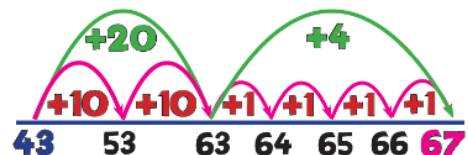
Explore that the ones digit does not change.

Complete a 10 using number bonds.



### A3: Forwards Jump

<sup>2</sup>  $43 + 24 = 67$



### A4: Partitioning

$$43 + 24 = 67$$

$$40 + 20 = 60$$

$$3 + 4 = 7$$

$$60 + 7 = 67$$

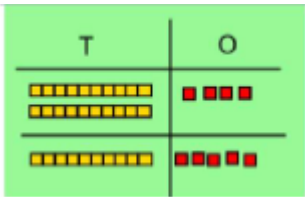
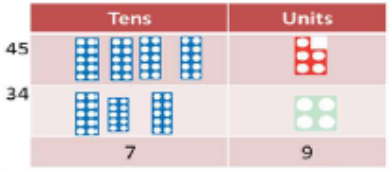
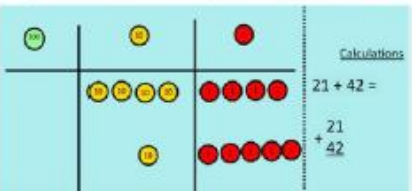
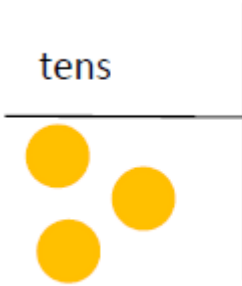
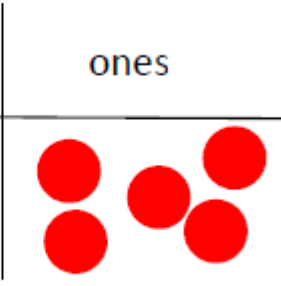
### A5: Partition Jot

$$43 + 24 = 67$$

$$60 + 7$$

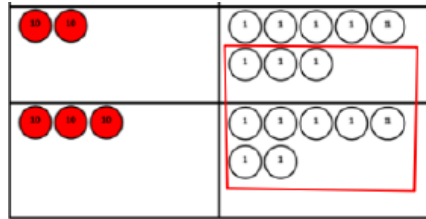
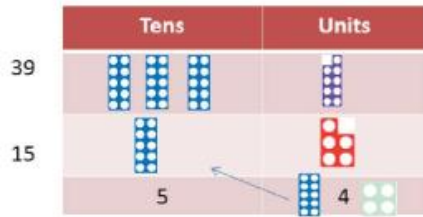


# Addition – Year 3

Objective/strategy	Concrete	Pictorial	Abstract
<p>Column Addition—no regrouping (friendly numbers).</p> <p>Add two or three 2 or 3digit numbers.</p> <p>Autumn 2</p>	<p>Lots of practise with concrete resources.</p> <p>Dienes:</p>  <p>Numicon:</p>  <p>Place value counters:</p> 	<p>Children should move onto drawing these representations into their books.</p>  	<p>Children should move onto the formal written method. Practising setting it out, leaving the line to carry numbers in future lessons.</p> $  \begin{array}{r}  223 \\  + 114 \\  \hline  337  \end{array}  $

Column addition with regrouping.

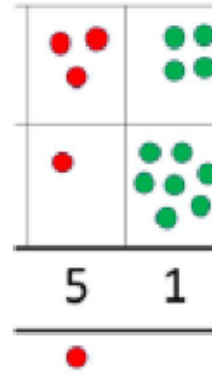
Autumn 2



Exchange ten ones for a ten. Model using Numicon and place value counters.

Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.

**DRAW MODEL FOR THIS**



Introduce expanded method if appropriate before moving onto the more formal method.

**A6a: <sup>Expanded</sup> Column Addition**

$$246 + 387 = 633$$

$$200 + 40 + 6$$

$$300 + 80 + 7$$

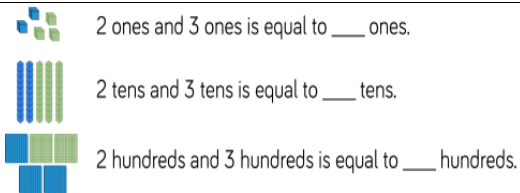
$$500 + 120 + 13$$

**A7c: Column Addition**

$$\begin{array}{r} 687 \\ + 248 \\ \hline 935 \end{array}$$

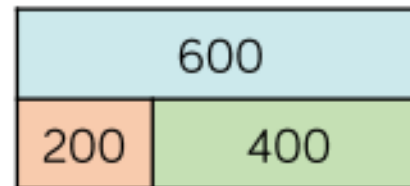
Multiples of 100 (can be combined with subtraction).

Autumn 2



Use similar models used in KS1.

Drawing bar models and part-part whole models.



Moving onto number sentences. Build relationships with inverse operations.

$$600 = 200 + 400$$

$$200 + 400 = 600$$

$$600 - 200 = 400$$

$$600 - 400 = 200$$

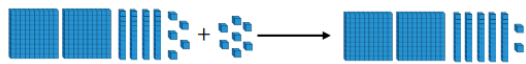
Adding to 3-digit numbers mentally or using jottings.

Autumn 2

$$176 + 40 =$$



$$245 + 7 =$$

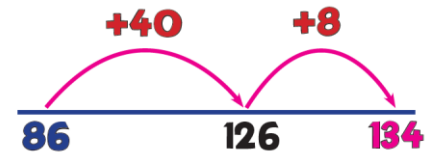


Use dienes and other concrete resources so children can explore and play with the calculations.

### A3b: Forwards Jump

<sup>2/3</sup>

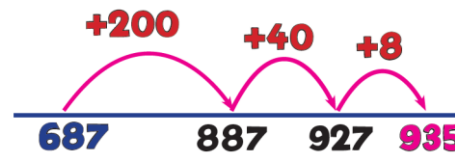
$$86 + 48 = 134$$



### A3c: Forwards Jump

<sup>3</sup>

$$687 + 248 = 935$$



### A4c: Partitioning

<sup>3</sup>

$$687 + 248 = 935$$

$$600 + 200 = 800$$

$$80 + 40 = 120$$

$$7 + 8 = 15$$

$$800 + 120 + 15 = 935$$

Children will likely show a preference between the pictorial (number lines) and the more abstract representations of partitioning. Both are acceptable forms of jotting.

Estimating answers and using inverse operations to check questions.

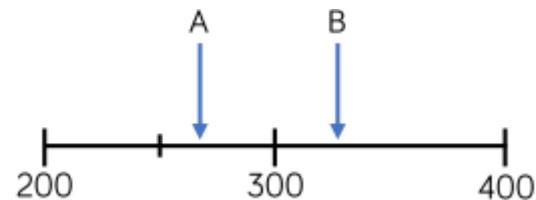
Autumn 2



Estimating  $98 + 17 = ?$

$$100 + 20 = 120$$

Use number lines to illustrate estimation.



Building up known facts and using them to illustrate the inverse and to check answers:

$$98 + 18 = 116 \quad 116 - 18 = 98$$

$$18 + 98 = 116 \quad 116 - 98 = 18$$

# Addition – Year 4 – 6

**Objective/strategy**  
**y**

**Concrete**

**Pictorial**

**Abstract**

Estimate and use inverse operations to check answers to a calculation.

Autumn 2

As per Year 3.

Building on the place value and level of calculations needed for your year group.

Y4 – column addition.

Autumn 2

Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.

$$3242 + 2213 =$$

1,000s	100s	10s	1s

$$3356 + 2435 =$$

Th	H	T	O

Draw place value grids into books.

Th	H	T	O

Th	H	T	O

Th	H	T	O

Th	H	T	O

## A7d: Column Addition

$$\begin{array}{r}
 4873 \\
 + 3762 \\
 \hline
 8635
 \end{array}$$

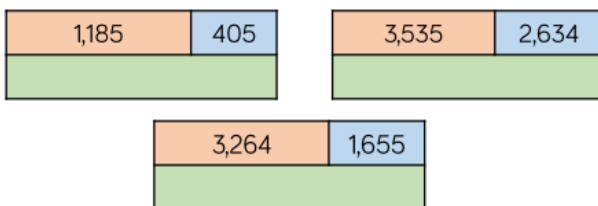
Move onto the formal written method, building on from what the children have done in Year 3. Use expanded method to begin with if necessary.



Y4 – 6 – mental methods, number lines and partitioning for addition.

Autumn 2

These are a range of other methods or representation that the children should be exposed to. Partitioning and number lines should be introduced in previous years and built upon now.



Bar models used to represent the calculations.

## A5d: Partition Jot

Partitioning jot – applying understanding of place value taught in previous blocks.

$$4873 + 3762 = 8635$$

$$7000 + 1500 + 130 + 5$$

Concrete resources used for previous methods can be used here to support children's conceptual understanding.

## MA1: Partitioning

MC RaPa CoDa Numbos  
4

$$648 + 231 = 879$$

$$800 + 70 + 9 = 879$$

## MA4b: Counting On

MC RaPa CoDa Numbos  
5

Thousands

$$7583 + 5000 = 12583$$

$$7583 + 5000 = 12583$$

## MA2: Round & Adjust

MC RaPa CoDa Numbos  
4

$$345 + 298 = 643$$

$$345 + 300 - 2$$

$$645 - 2 = 643$$



Y5—add numbers with more than 4 digits.

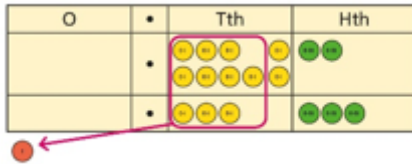
Autumn 2

Add decimals with 2 decimal places, including money.

Y5 Summer 1  
Y6 Spring 1

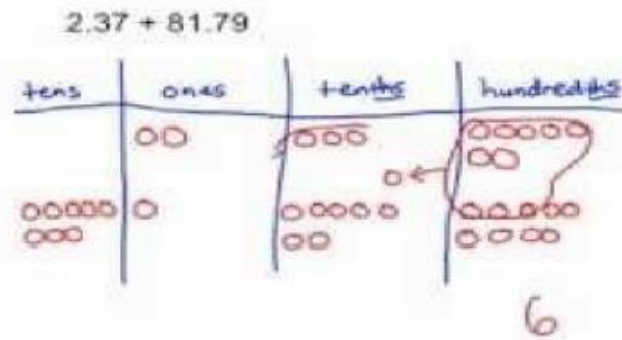
As Year 4

$$0.92 + 0.33 =$$



Introduce decimal place value counters to model exchanges.

Draw place value charts and counters in maths books to support.



### A7h: Column Addition

$$\begin{array}{r} 76.7 \\ + 58.5 \\ \hline 135.2 \end{array}$$

Use alongside measures, including money.

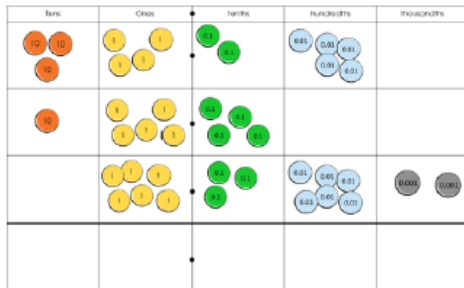
### A7i: Column Addition With Money

$$\begin{array}{r} \text{€}38.25 \\ + \text{€}27.46 \\ \hline \text{€}65.71 \end{array}$$

Y6—add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points.

Spring 1

As Year 5



As Year 5

Insert 0s for place holders.

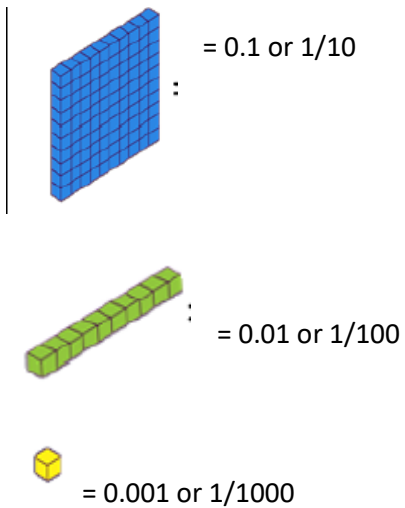
### A7j: Column Addition With Decimals

$$\begin{array}{r} 73.4 \\ + 5.67 \\ \hline 79.07 \end{array}$$

Year 4 – 6 mental calculations (decimal numbers).

Y4 Spring 4  
Y5 Summer 1  
Y6 Spring 1

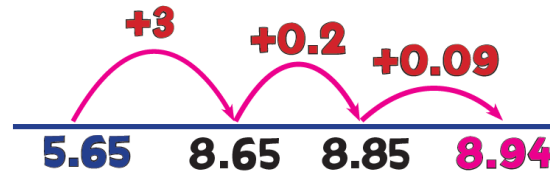
Use dienes to represent decimal numbers. Place value counters can also be used.



Draw number lines as jottings. Relate these to money and measure where appropriate.

### A3g: Decimal Jump

$$5.65 + 3.29 = 8.94$$



Continue to use number lines to solve decimal calculations. Children should eventually be able to use the number line 'in their head' when confident with the place value.

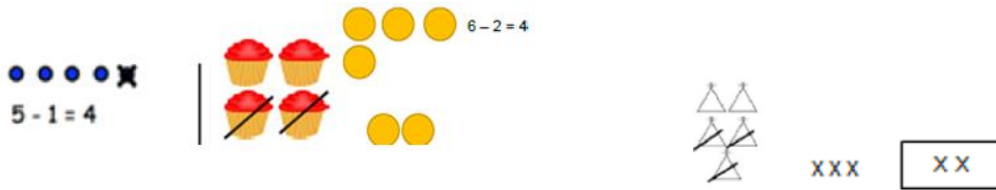
Some children may prefer partitioning so this could be an alternative method for the children to use.

### A5i: Partition Jot

$$\pounds 38.25 + \pounds 27.46 = \pounds 65.71$$
$$\pounds 65.00 + \pounds 0.71$$

# Subtraction – EYFS

Concrete apparatus and real life examples used to relate subtraction to taking away and counting how many objects are left.



Children then begin to use pictorial representations.

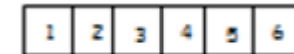
Children are encouraged to read number sentences aloud in different ways. E.g “five subtract one leaves four” and “four is equal to five subtract one”. Construct number sentences verbally or with cards to support pictorial and concrete representations.

Children made a record in pictures, words or symbols of subtraction activities already carried out.



Solve simple problems using fingers.

Number tracks can be introduced to find one less than any given number. E.g what is one less than 9 or one less than 20.





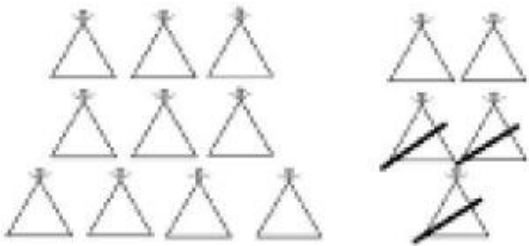


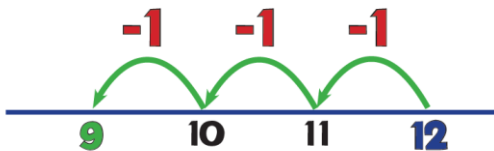
Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back under the number line.



Children will need opportunities to look at and talk about different models and images as they move between representations.



# Subtraction – Year 1

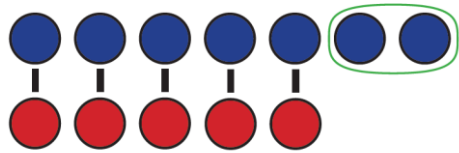
Objective/strategy	Concrete	Pictorial	Abstract
<p>Taking away ones.</p> <p>Autumn 2 Spring 1</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p><b>S1: Objects</b></p>  <p><b>7 - 3 = 4</b></p> <p><small>"What do I get if I take 3 away from 7? Answer: 4"</small></p> <p><b>6 - 4 = 2</b></p> 	<p>Cross out drawn objects to show what has been taken away.</p>  <p><b>15 - 3 = 12</b></p>	<p>Move onto abstract number sentences, only taking away ones.</p> <p><b>7 - 4 = 3</b></p> <p><b>16 - 9 = 7</b></p>
<p>Counting back.</p> <p>Autumn 2 Spring 1</p>	 <p>Move objects away from the group, counting backwards.</p>  <p>Move the beads along the bead string as you count backwards.</p>	<p><b>S3a: Counting Back</b></p>  <p><b>12 - 3 = 9</b></p> <p><small>"What do I get if I take 3 away from 12? Answer: 9"</small></p>	<p>Put 13 in your head, count back 4. What number are you at?</p>

Find the difference.

Autumn 2

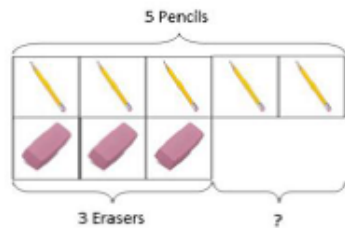
Compare objects and amounts.

### S2: What's the Difference?



$$7 - 5 = 2$$

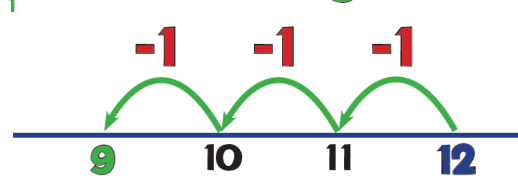
"How many more is 7 than 5? What is the difference?"



The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

Count on or back using a number line to find the difference.

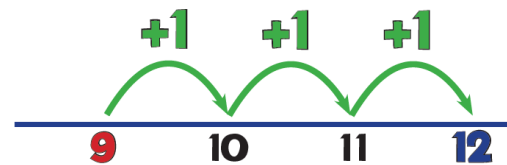
### S3a: Counting Back



$$12 - 3 = 9$$

"What do I get if I take 3 away from 12? Answer: 9"

### S4: Counting On



$$12 - 9 = 3$$

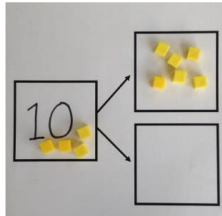
"How many more is 12 than 9? What is the difference?"

Children understand 'find the difference' as subtraction.

Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?

Represent and use number bonds and related subtraction facts within 20.

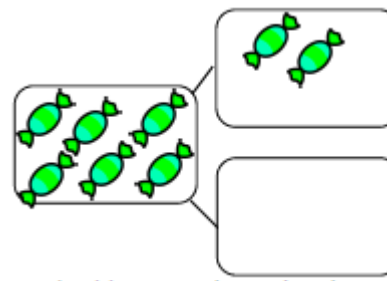
Autumn 2  
Spring 1



Link to addition. Use part-part whole model to model the inverse.

If 10 is the whole and 6 is one of the parts, what is the other part?

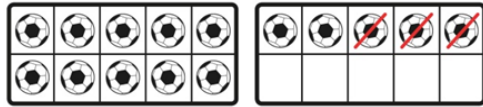
$$10 - 6 = 4$$



Use pictorial representations to show the part.

Move to using numbers within the part whole model.



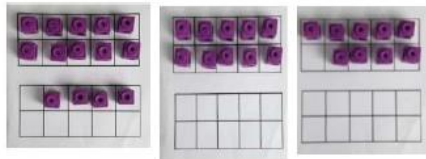


$$15 - 3 = 12$$

Use tens frames to support subtraction.

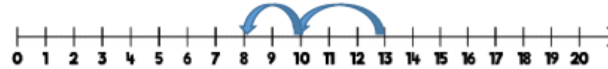
Make 10.  
Spring 1

$$14 - 5 =$$



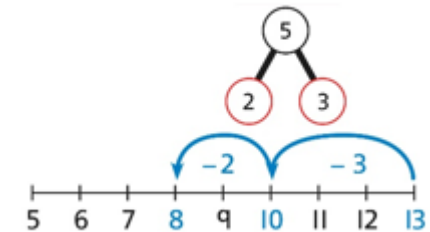
Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.

Jump back 3 first, then another 2. Use ten as the stopping point.



How many do we take off first to get to 10?

$$13 - 5$$



Bar models  
Autumn 2  
Spring 1



$$6 - 4 = 2$$



Begin to draw models to help represent calculations.



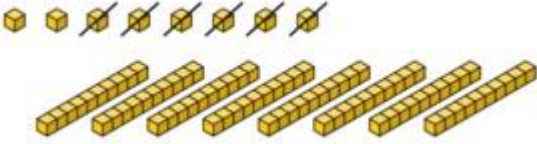
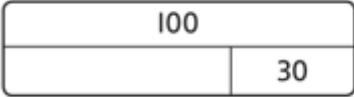
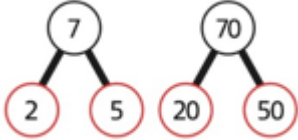
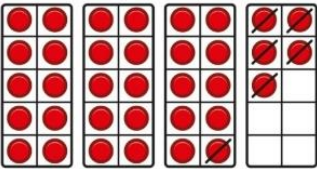
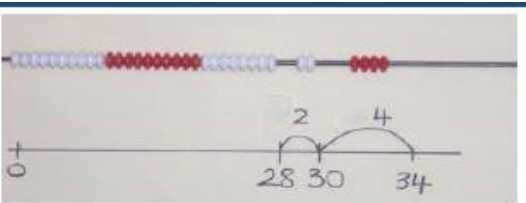
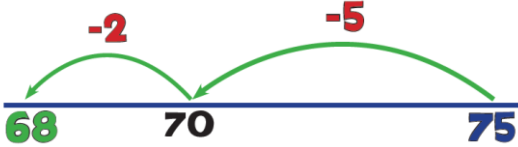
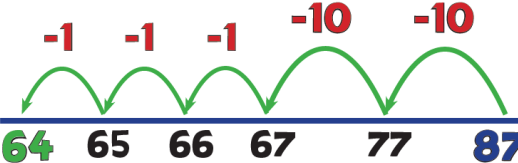

$$10 = 8 + 2$$

$$10 = 2 + 8$$

$$10 - 2 = 8$$

$$10 - 8 = 2$$

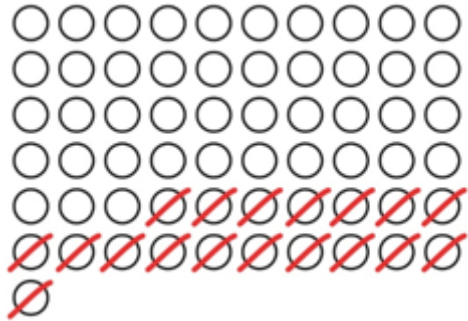
# Subtraction – Year 2

Objective/strategy	Concrete	Pictorial	Abstract
<p>Subtracting multiples of 10</p> <p>Autumn 2</p>	<p>Use known number bonds and unitising to subtract multiples of 10.</p>  <p>8 subtract 6 is 2. So, 8 tens subtract 6 tens is 2 tens.</p>	<p>Use known number bonds and unitising to subtract multiples of 10.</p>  <p><math>10 - 3 = 7</math> So, 10 tens subtract 3 tens is 7 tens.</p>	<p>Use known number bonds and unitising to subtract multiples of 10.</p>  <p>7 tens subtract 5 tens is 2 tens. <math>70 - 50 = 20</math></p>
<p>Subtracting by bridging 10.</p> <p>Autumn 2</p>	<p>Use concrete objects and representations such as tens frames.</p> <p><math>35 - 6 = 29</math></p>  <p>I took away 5 counters, then 1 more.</p>  <p>Use a bead spring to model</p>	<p>Use number lines to support subtractions.</p> <p><b>S5a: Backwards Boing</b> 2</p>  <p><b>75 - 7 = 68</b></p> <p><b>S6a: Backwards Bounce</b> 2</p>  <p><b>87 - 23 = 64</b></p>	<p>Continue to use number lines to support calculating subtractions when bridging 10.</p> <p>Children who are confident should use the number in their head or use simple jottings to support calculations.</p> <p><b>MS1: Counting Back</b></p> <p><b>46 - 21 = 25</b></p> 

Subtracting – other strategies for taking away 2-digit numbers.

Autumn 2

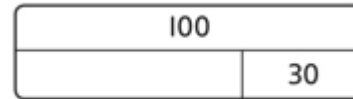
Subtract by taking away.



$$61 - 18 =$$

Subtract the 10s and the 1s.

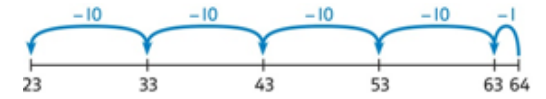
This can be represented on a 100 square or a bar model.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Subtract the 10s and the 1s.

This can be represented on a number line.

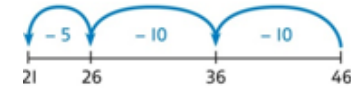


$$64 - 41 = ?$$

$$64 - 1 = 63$$

$$63 - 40 = 23$$

$$64 - 41 = 23$$



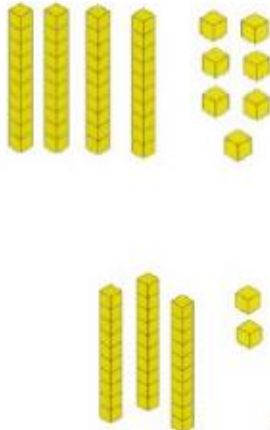
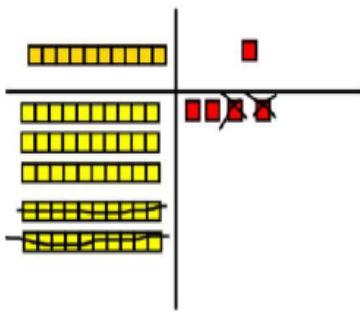
$$46 - 20 = 26$$

$$26 - 5 = 21$$

$$46 - 25 = 21$$



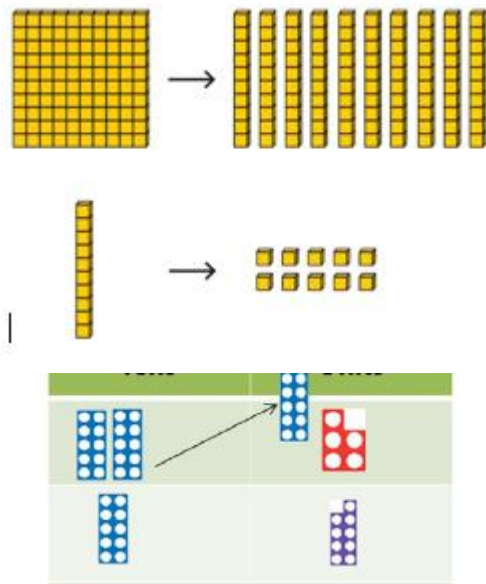
# Subtraction – Year 3

Objective/strategy	Concrete	Pictorial	Abstract
<p>Column subtraction without regrouping.</p> <p>Autumn 2</p>	<p>Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away.</p> <p><math>47 - 22 =</math></p> 	<p>Draw representations in maths books to support working out. Continue to use place value equipment such as dienes or counters.</p>  <p style="text-align: right;">Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$	<p>Introduce the formal method, beginning with the expanded method.</p> <p><b>S10: Expanded Column</b>  <small>2 Additional Subtraction</small></p> $87 - 23 = 64$ $\begin{array}{r} 80 & 7 \\ 20 & 3 \\ \hline 60 & 4 \end{array}$ <p>When children are confident with the place value, move onto the compact method.</p> <p><b>S11: Column Subtraction</b>  <small>2 Additional</small></p> $\begin{array}{r} 87 \\ - 23 \\ \hline 64 \end{array}$

Column subtraction with regrouping.

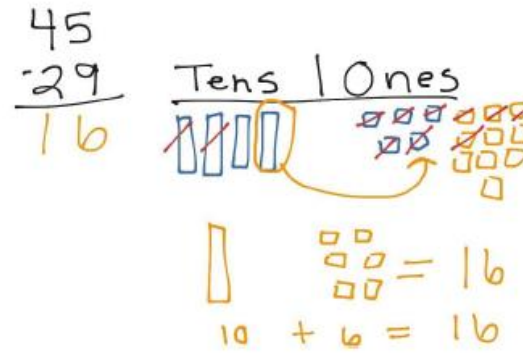
Autumn 2

Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones.



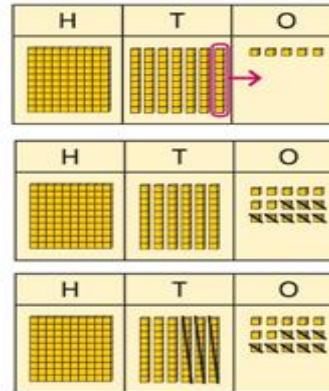
Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones.

Allow children to draw the equipment as pictures in their maths books.



$$175 - 38 = ?$$

I need to subtract 8 ones, so I will exchange a ten for 10 ones.



Introduce the formal method, beginning with the expanded method.

### S10: Expanded Column Subtraction

$$75 - 37 = 38$$

$$\begin{array}{r} 60 & & 15 \\ & \cancel{70} & 5 \\ & 30 & 7 \\ \hline & 30 & 8 \end{array}$$

When children are confident with the place value, move onto the compact method.

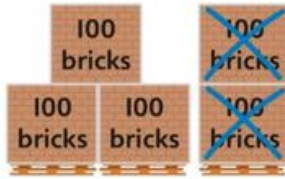
### S11: Column Subtraction

$$\begin{array}{r} 6 & 11 & 1 \\ & \cancel{72} & 3 \\ - & 35 & 6 \\ \hline & 36 & 7 \end{array}$$

Subtracting multiples of 100.

Autumn 2

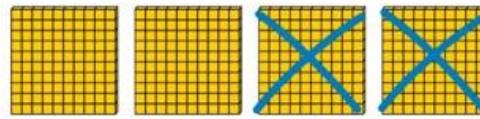
Use known facts and unitising to subtract multiples of 100.



$$5 - 2 = 3$$

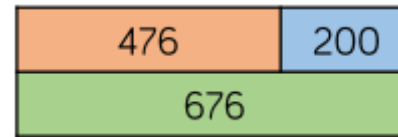
$$500 - 200 = 300$$

Use images and bar models to represent the calculations. Link to known facts to support understanding.

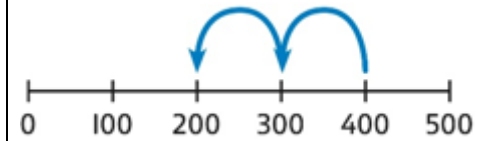


$$4 - 2 = 2$$

$$400 - 200 = 200$$



Understand the link with counting back in 100s.

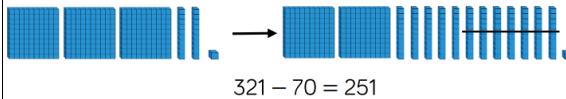


Use known facts and unitising as efficient and accurate methods. I know that  $7 - 4 = 3$ . Therefore, I know that  $700 - 400 = 300$ .

Subtracting numbers mentally.

Autumn 2

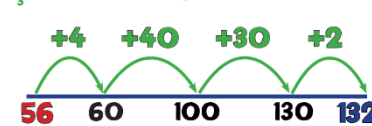
Use concrete objectives to show what is happening when subtracting.



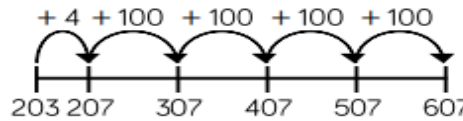
$$321 - 70 = 251$$

Children should be encouraged to use jottings to solve subtractions mentally using number bonds:

**S8b: Frog**

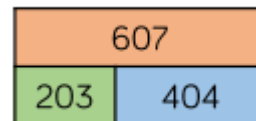


$$132 - 56 = 76$$



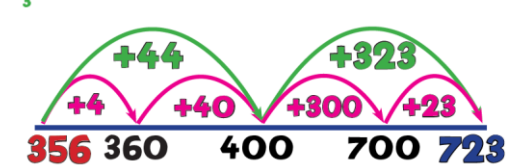
Counting on:

Bar models to represent the calculations:



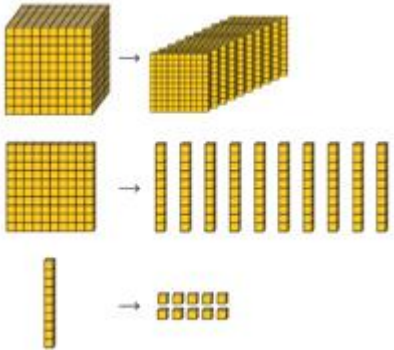
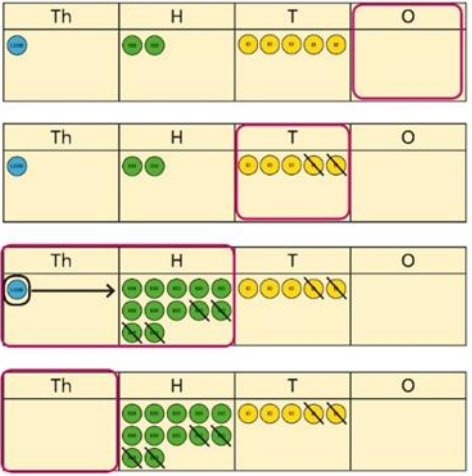
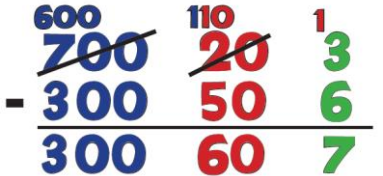
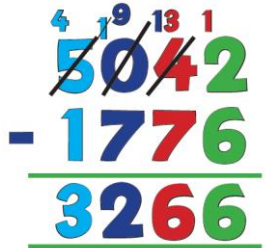
Children should continue to explore mental methods and use jottings to support these with more complex subtractions.

**S8c: Frog**



$$723 - 356 = 367$$

# Subtraction – Year 4

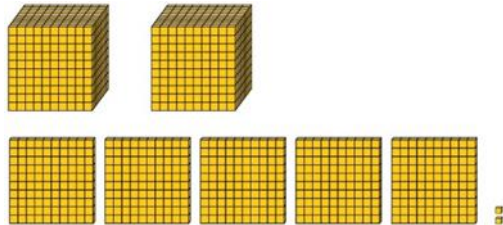
Objective/strategy	Concrete	Pictorial	Abstract
<p>Column subtraction with exchanging.</p> <p>Autumn 2</p>	<p>Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.</p> 	<p>Represent place value equipment on a place value grid to subtract, including exchanges where needed.</p> 	<p>Introduce the formal method, beginning with the expanded method if necessary.</p> <p><b>S10: Expanded Column</b>  <small>Subtraction (100, 10, 1s)</small></p> $723 - 356 = 367$  <p>Children should have lots of opportunities to practice column subtraction with exchanges.</p> <p><b>S11d: Column Subtraction</b></p> 

Column subtraction with exchanges across more than one column.

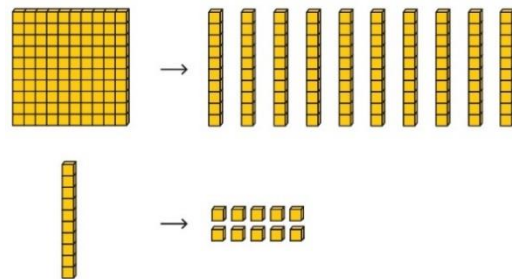
Autumn 2

Understand why two exchanges may be necessary.

$$2,502 - 243 = ?$$

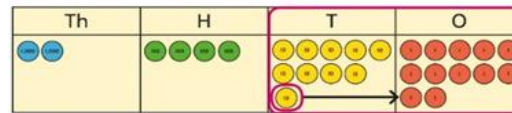
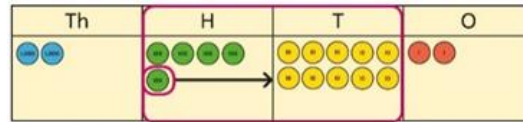


*I need to exchange a 10 for some 1s, but there are not any 10s here.*



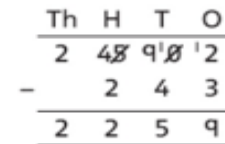
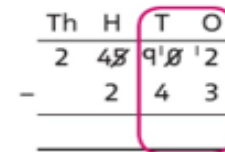
Make exchanges across more than one column where there is a zero as a place holder.

$$2,502 - 243 = ?$$



Make exchanges across more than one column where there is a zero as a place holder.

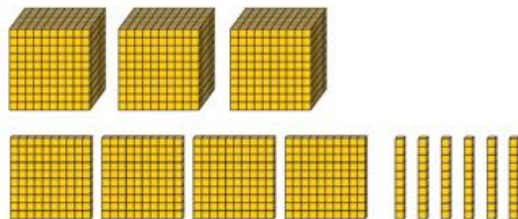
$$2,502 - 243 = ?$$



Subtracting numbers mentally.

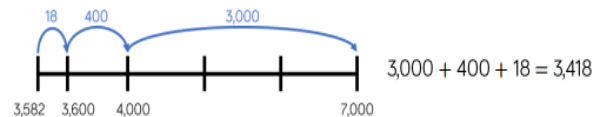
Autumn 2

Continue to use place value equipment to support children with solving subtraction problems.

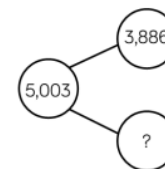
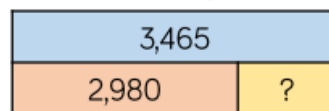


*What number will be left if we take away 300?*

Provide children with opportunities to solve subtractions where they need to take away, find the difference or counting on.

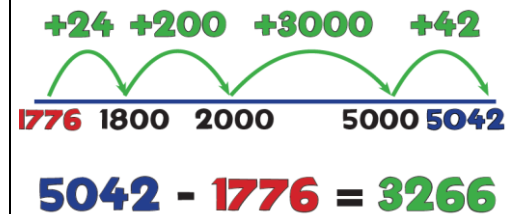


Use bar models and part-part whole models.

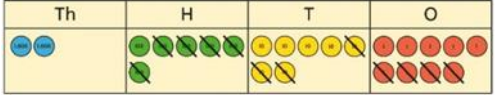

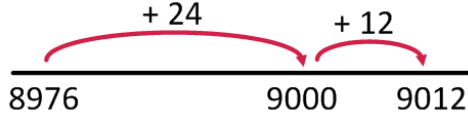


Continue to encourage children to find the most efficient method (mental or written) and use jottings where appropriate.

**S8d Frog**



# Subtraction – Year 5/6

Objective/strategy	Concrete	Pictorial	Abstract
Column subtraction and mental methods.  Autumn 2	As LKS2 – continue to build on these methods, using larger numbers as specified in the National Curriculum.		
Selecting efficient methods for subtraction.  Autumn 2	Continue to use place value equipment used by LKS2 to support understanding of different strategies.  	Using counting on to find the difference (useful when the numbers are close together).  $75\,221 - 14\,300$    $9012 - 8976$    Use round and adjust e.g.  $122,456 - 11,999 =$  Round $122,456 - 12,000 = 110,456$  Adjust $110,456 + 1 = 110,457$	





## **Multiplication and Division**

### **Key Stage One**

Children develop an awareness of equal groups and link this with counting in equal steps, starting with 2s, 5s and 10s. In Year 2, they learn to connect the language of equal groups with the mathematical symbols for multiplication and division. They learn how multiplication and division can be related to repeated addition and repeated subtraction to find the answer to the calculation. In this key stage, it is vital that children explore and experience a variety of strong images and manipulative representations of equal groups, including concrete experiences as well as abstract calculations.

Children begin to recall some key multiplication facts, including doubles, and an understanding of the 2, 5 and 10 times tables and how they are related to counting.

### **Lower Key Stage Two**

Children build a solid grounding in times tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35.

Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively.

Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit. Children develop column methods to support multiplications in these cases.

For successful division, children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.

### **Upper Key Stage Two**

Building on their understanding, children develop methods to multiply up to 4-digit numbers by single-digit and 2-digit numbers.

Children develop column methods with an understanding of place value, and they continue to use the key skill of unitising to multiply and divide by 10, 100 and 1,000.

Written division methods are introduced and adapted for division by single-digit and 2-digit numbers and are understood alongside the grid method and place value.

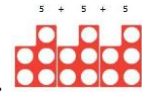
Multiplication and division of decimals are also introduced and refined in Year 6.



# Multiplication – EYFS

Although there is no explicit object which refers to multiplication in EYFS, the foundations of multiplication will first be introduced here. The link between addition and multiplication is first of all introduced through doubling .

If available, Numicon should be used to introduce repeated addition of the same number and recorded through pictorial representations.



Children may begin to record this pictorially.



e.g. how many groups of 2 are there?

Real life contexts and use of practical equipment can be used to count in repeated groups of the same size.



e.g. How many wheels are there altogether. How much money do I have?




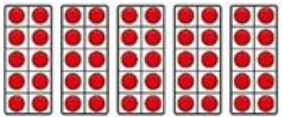

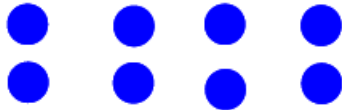
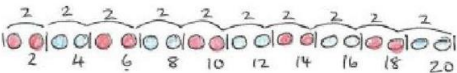
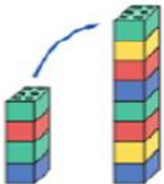





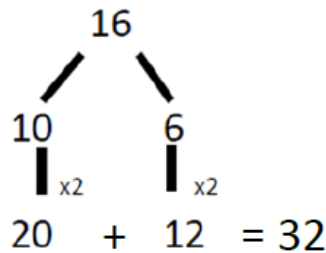
Begin to count in 2s and 5s and 10s aloud and with objects.



Children are given multiplication problems in real life contexts. Children are encouraged to visualize the problem.

e.g. How many fingers have you got on two hands? How many sides would there be on 2 triangles? How many legs would there be on four ducks?

# Multiplication – Year 1

Objective/strategy	Concrete	Pictorial	Abstract
<p>Counting in multiples (2s, 5s and 10s).</p> <p>Summer 1</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p>  <p><math>4 \times 5 =</math></p>  <p><math>4 \times 2 =</math></p>  <p>There are 5 pens in each pack ... 5...10...15...20...25...30...35...40...</p>	<p>Use 100 squares and ten frames support counting in 2s, 5s and 10s.</p>   <p>Children make representations to show counting in multiples.</p>  	<p>Describe equal groups using words.</p> <p>Three equal groups of 4. Four equal groups of 3.</p> <p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p>
<p>Doubling</p> <p>Summer 1</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling.</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>   	<p>Draw pictures to show how to double numbers.</p> <p>Double 4 is 8.</p>  	<p>Partition a number and then double each part before recombining it back together.</p> 

Making equal groups and counting them.

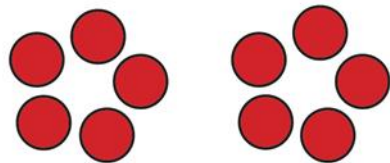
Summer 1

Children arrange objects in equal and unequal groups and understand how to recognise whether they are equal.



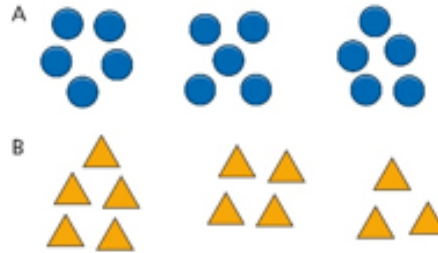
Use manipulatives to create equal groups.

### M1: Groups



"2 groups of 5 counters makes 10 counters altogether"

Children draw and represent equal and unequal groups.



Describe equal groups using words.

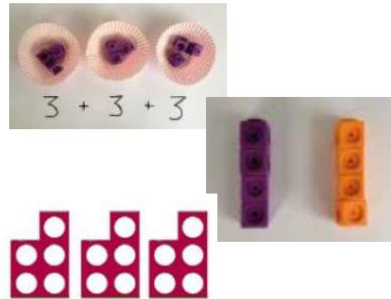
Three equal groups of 4.

Four equal groups of 3.

Repeated addition

Summer 1

Use different objects to add equal groups.



Children draw images of equal groups to support repeated addition.

### M1: Repeated Addition

(Groups)

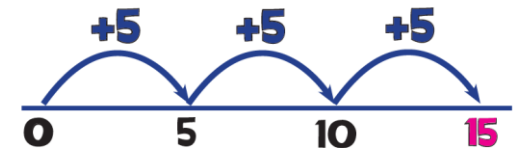


$$3 \times 5 = 5 + 5 + 5 = 15$$

Use number lines to show repeated addition.

### M2: Repeated Addition

(Number Line)



$$3 \times 5 = 5 + 5 + 5 = 15$$

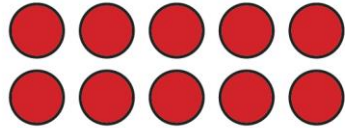
"5 times 3" means "5, 3 times!"

Using arrays

Summer 1

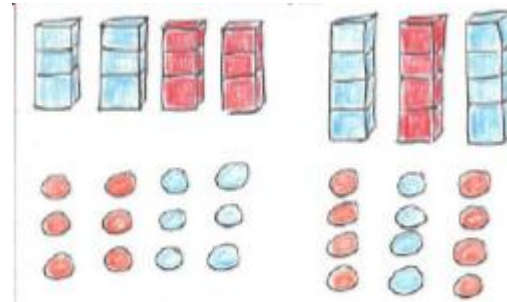
Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.

### M3: Arrays



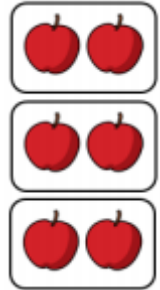
"2 groups of 5 counters" or "5 groups of 2 counters" - "10 counters altogether"

Draw pictures of arrays to support understanding.





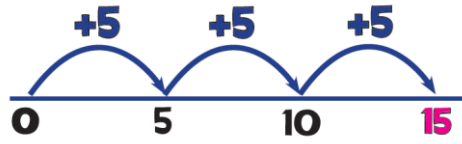

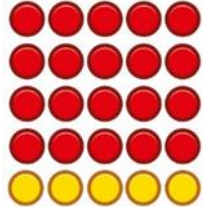
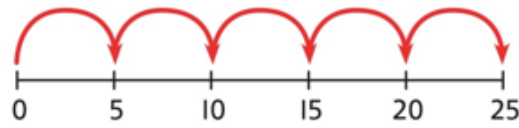
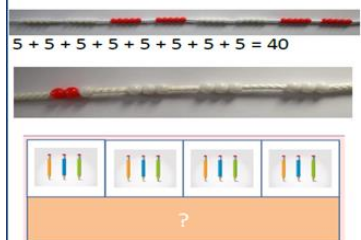
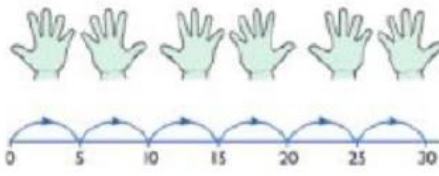
Draw and describe arrays.

There are \_\_\_\_ apples in each row. There are \_\_\_\_ rows. \_\_\_\_ + \_\_\_\_ + \_\_\_\_ = \_\_\_\_ There are \_\_\_\_ apples altogether.



There is no need for children to know, understand or recognise the multiplication symbol at this stage. These methods are essential prerequisites for multiplication from Year 2 onwards.

# Multiplication – Year 2

Objective/strategy y	Concrete	Pictorial	Abstract
<p>Equal groups and repeated addition.</p> <p>Autumn 4 Spring 1</p>	<p>Recognise equal groups and write as repeated addition and as multiplication.</p>  <p>3 groups of 5 chairs 15 chairs altogether</p>	<p>Recognise equal groups using standard objects such as counters and write as repeated addition and multiplication.</p>  <p>3 groups of 5 15 in total</p>	<p>Use a number line and write as repeated addition and as multiplication.</p> <p><b>M2: Repeated Addition</b> <small>(Number Line)</small></p>  <p><b>3 x 5 = 5 + 5 + 5 = 15</b> <small>"5 times 3" means "5, 3 times!"</small></p>
<p>Using arrays to represent multiplication and support understanding.</p> <p>Autumn 4 Spring 1</p>	<p>Understand the relationship between arrays, multiplication and repeated addition.</p>  <p>4 groups of 5</p>	<p>Draw and use arrays to support understanding.</p>  <p>4 groups of 5 ... 5 groups of 5</p>	<p>Continue to use number lines to solve multiplication calculations.</p>  <p><math>5 \times 5 = 25</math></p>
<p>Counting in multiples of 2, 3, 4, 5, 10 from 0.</p> <p>Autumn 4 Spring 1</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p> 	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p>

Learning  $\times 2$ ,  $\times 5$  and  $\times 10$  table facts.

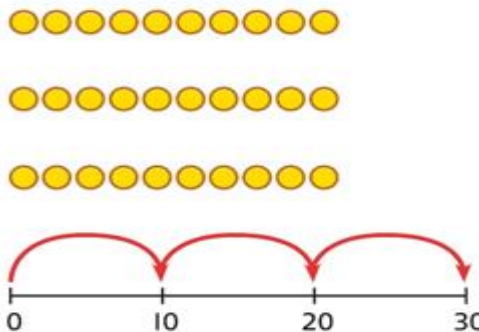
Autumn 4  
Spring 1

Develop an understanding of how to unitise groups of 2, 5 and 10 and learn corresponding times-table facts.



3 groups of 10 ... 10, 20, 30  
 $3 \times 10 = 30$

Continue to use images and number lines to support understanding of multiplication facts.



$$10 + 10 + 10 = 30$$
$$3 \times 10 = 30$$

Record as number sentences, showing understanding of the multiplication symbol.

$$2 \times 5$$

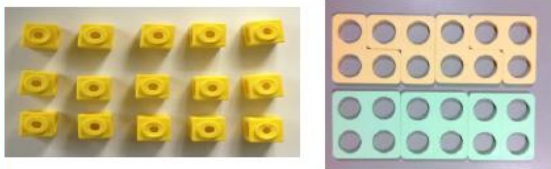
$$3 \times 2$$

$$10 \times 5$$

Understanding commutativity.

Autumn 4  
Spring 1

Use arrays to visualise commutativity.



Form arrays using counters to visualise commutativity. Rotate the array to show that orientation does not change the multiplication.




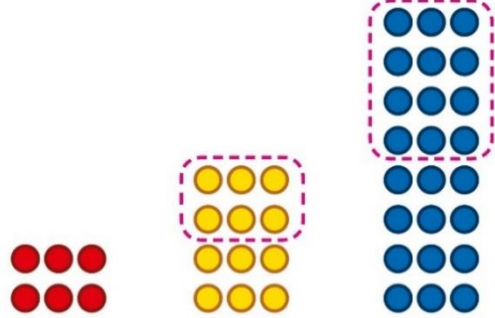
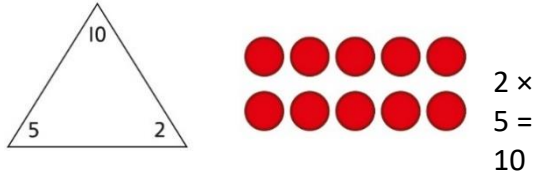
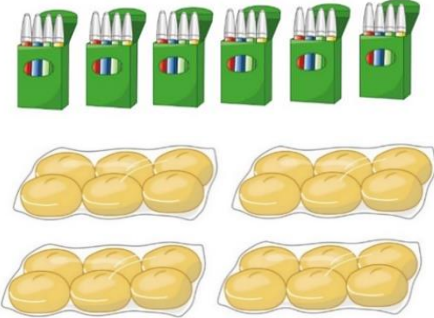
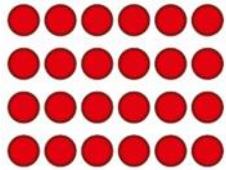
This is 2 groups of 6 and also 6 groups of 2.

$$10 = 2 \times 5$$

$$10 = 5 \times 2$$



# Multiplication – Year 3

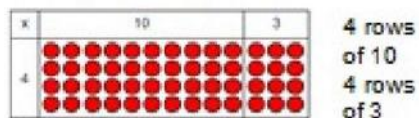
Objective/strategy y	Concrete	Pictorial	Abstract
<p>Understanding and using <math>\times 3</math>, <math>\times 2</math>, <math>\times 4</math> and <math>\times 8</math> tables.</p> <p>Autumn 3 Spring 1</p>	<p>Children learn the times-tables as ‘groups of’, but apply their knowledge of commutativity.</p> 	<p>Children should be able to draw a range of diagrams to represent the multiplication facts, such as arrays.</p> <p>Children understand how the <math>\times 2</math>, <math>\times 4</math> and <math>\times 8</math> tables are related through repeated doubling.</p>  <p><math>3 \times 2 = 6</math>      <math>3 \times 4 = 12</math>      <math>3 \times 8 = 24</math></p>	<p>Children should be able to quickly recall these key multiplication facts and relate them to division.</p>  <p><math>2 \times 5 = 10</math> <math>5 \times 2 = 10</math> <math>10 \div 5 = 2</math> <math>10 \div 2 = 5</math></p>
<p>Using commutativity to support understanding of the times-tables.</p> <p>Autumn 3 Spring 1</p>	<p>Understand how to use times-tables facts flexibly.</p>  <p>There are 6 groups of 4 pens. There are 4 groups of 6 bread rolls. I can use <math>6 \times 4 = 24</math> to work out both totals.</p>	<p>Understand how times-table facts relate to commutativity.</p> <p><math>6 \times 4 = 24</math> <math>4 \times 6 = 24</math></p> 	<p>Understand how times-table facts relate to commutativity.</p> <p><i>I need to work out 4 groups of 7.</i></p> <p><i>I know that <math>7 \times 4 = 28</math></i></p> <p><i>so, I know that</i></p> <p><i>4 groups of 7 = 28</i> <i>and</i> <i>7 groups of 4 = 28.</i></p>

Multiplying a 2-digit number by a 1-digit number (grid method).

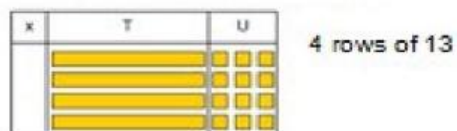
Formal written method for multiplication.

Autumn 3  
Spring 1

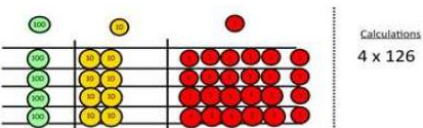
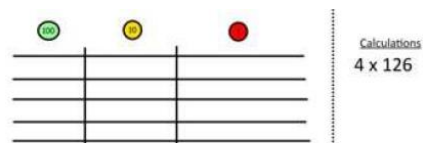
Show the links with arrays to first introduce the grid method.



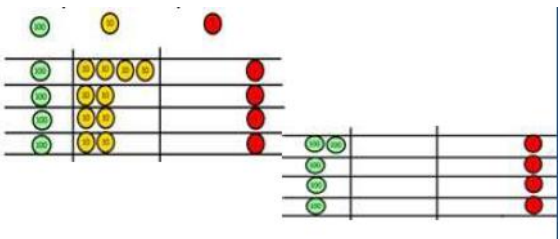
Move onto base ten to move towards a more compact method.



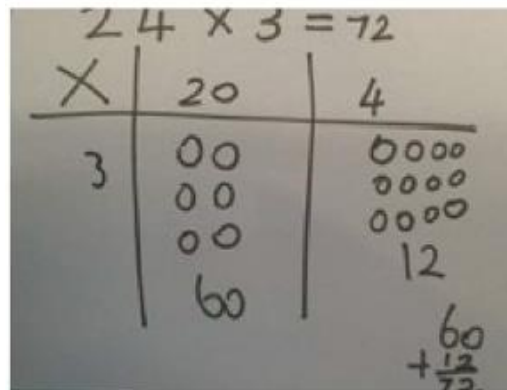
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows:



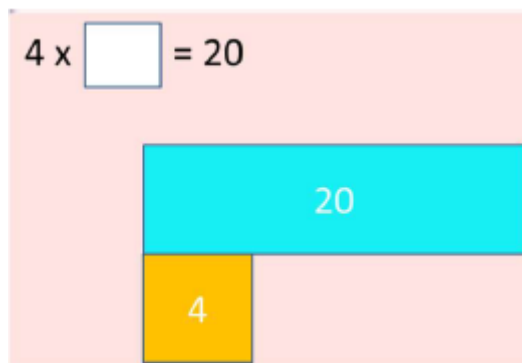
Fill each row with 126.  
Add up each column, starting with the ones making any exchanges needed  
Then you have your answer.



Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model are used to explore missing numbers.



Use the grid method as a written method for multiplication.

### M5a: Grid Method Short Multiplication

$$43 \times 6 = 258$$

x	40	3	240
6	240	18	+ 18
			<u>258</u>

If children are ready, the formal written method for multiplication should be introduced.

### M7: Column Multiplication 3 Additional

$$\begin{array}{r} 15 \\ \times 5 \\ \hline 75 \end{array}$$



Mental methods for multiplication.

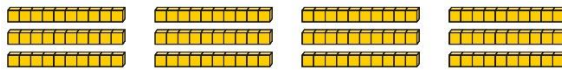
Autumn 3  
Spring 1

Explore the relationship between known times-tables and multiples of 10 using place value equipment.

Make 4 groups of 3 ones.

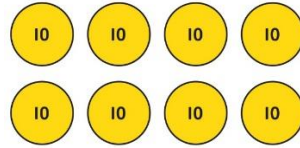
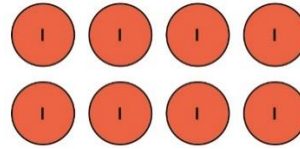


Make 4 groups of 3 tens.



What is the same?  
What is different?

Understand how unitising 10s supports multiplying by multiples of 10.



4 groups of 2 ones is 8 ones.  
4 groups of 2 tens is 8 tens.

$$4 \times 2 = 8$$
$$4 \times 20 = 80$$

Use known facts:

$$4 \times 2 = 8$$
$$4 \times 20 = 80$$

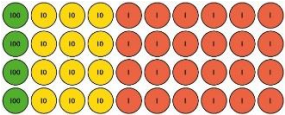
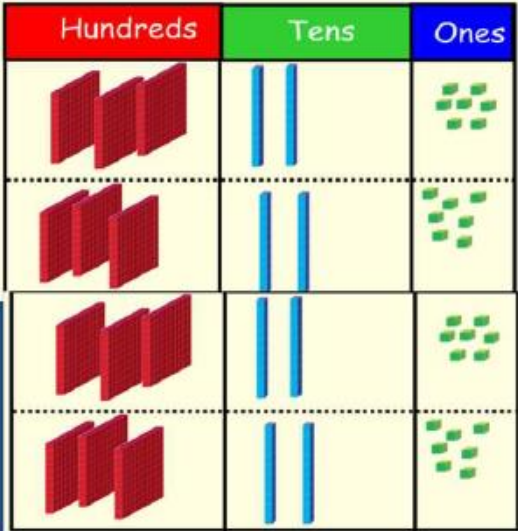
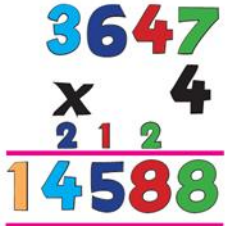
Partitioning using known facts:

## MM3: Partitioning

$$15 \times 5 = 75$$

$$\begin{array}{c} \text{50} \\ (10 \times 5) \end{array} + \begin{array}{c} \text{25} \\ (5 \times 5) \end{array} = 75$$

# Multiplication – Year 4

Objective/strategy	Concrete	Pictorial	Abstract										
<p>Column multiplication</p> <p>Autumn 4 Spring 1</p>	<p>Use place value equipment to make multiplications.</p> <p><i>Make <math>4 \times 136</math> using equipment.</i></p>  <p><i>I can work out how many 1s, 10s and 100s.</i></p> <p><i>There are <math>4 \times 6</math> ones... 24 ones</i>  <i>There are <math>4 \times 3</math> tens ... 12 tens</i>  <i>There are <math>4 \times 1</math> hundreds ... 4 hundreds</i></p> <p><math>24 + 120 + 400 = 544</math></p>	<p>Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.</p> <p><math>327 \times 4 =</math></p> 	<p>If necessary, teach short multiplication alongside the grid method to demonstrate what is happening/link to previous learning.</p> <p><b>M5b: Grid Method</b> Short Multiplication</p> <p><math>147 \times 4 = 588</math></p> <table border="1" data-bbox="1608 561 1953 671"> <tr> <td>x</td> <td>100</td> <td>40</td> <td>7</td> <td></td> </tr> <tr> <td>4</td> <td>400</td> <td>160</td> <td>28</td> <td></td> </tr> </table> <p style="text-align: right;"> <math>\begin{array}{r} 400 \\ 160 \\ + 28 \\ \hline 588 \end{array}</math> </p> <p>Children should be confident in multiplying 3 and 4 digit numbers by 1 digit numbers.</p> <p><b>M7a: Column Multiplication</b></p> 	x	100	40	7		4	400	160	28	
x	100	40	7										
4	400	160	28										

Multiplying more than two numbers (also applicable to Years 5 and 6).

Autumn 4  
Spring 1

Represent situations by multiplying three numbers together.



Each sheet has  $2 \times 5$  stickers.

There are 3 sheets.

There are  $5 \times 2 \times 3$  stickers in total.

$$5 \times 2 \times 3 = 30$$

$$\underbrace{\quad\quad\quad}_{10} \times 3 = 30$$

Understand that commutativity can be used to multiply in different orders.



$$2 \times 6 \times 10 = 120$$

$$12 \times 10 = 120$$

$$10 \times 6 \times 2 = 120$$

$$60 \times 2 = 120$$

Children should be encouraged to re-order a calculation so that they can use known facts to solve it.

## MM2: Re-ordering

$$(9 \times 2) \times 5$$

$$18 \times 5 = 90$$

$$(9 \times 5) \times 2$$

$$45 \times 2 = 90$$

$$(2 \times 5) \times 9$$

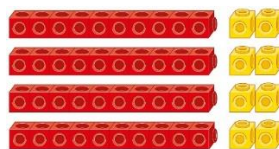
$$10 \times 9 = 90 *$$

Understanding and using partitioning in multiplication.

Autumn 4  
Spring 1

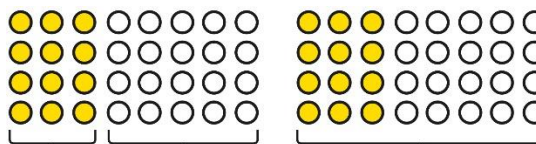
Make multiplications by partitioning.

$4 \times 12$  is 4 groups of 10 and 4 groups of 2.



$$4 \times 12 = 40 + 8$$

Understand how multiplication and partitioning are related through addition.



$$4 \times 3 = 12 \quad 4 \times 5 = 20$$

$$4 \times 8 = 32$$

$$4 \times 3 = 12$$

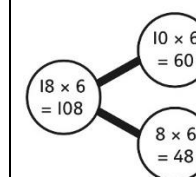
$$4 \times 5 = 20$$

$$12 + 20 = 32$$

$$4 \times 8 = 32$$

Use partitioning to multiply 2-digit numbers by a single digit.

$$18 \times 6 = ?$$



$$18 \times 6 = 10 \times 6 + 8 \times 6$$

$$= 60 + 48$$

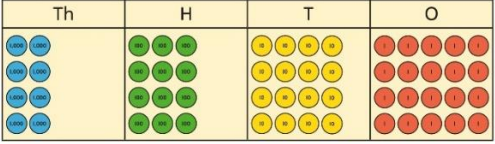
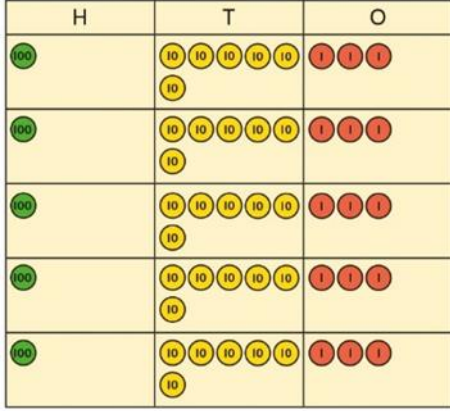


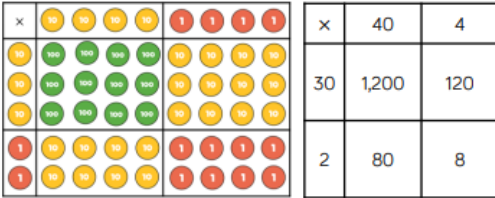
$$= 108$$

$$18 \times 6 = 10 \times 6 + 8 \times 6$$

$$= 60 + 48$$

$$= 108$$

# Multiplication – Year 5 – 6

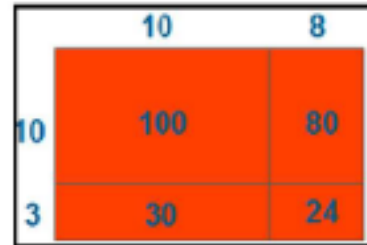
Objective/strategy	Concrete	Pictorial	Abstract																																						
<p>Short multiplication</p> <p>Y5 – Spring 1</p> <p>Y6 – Autumn 2</p>	<p>Consolidate work from Year 4. Use place value equipment to build understanding. Use equipment to explore multiplications.</p>  <p>4 groups of 2,345</p> <p>This is a multiplication:</p> $4 \times 2,345$ $2,345 \times 4$	<p>Represent multiplications using place value equipment and add the 1s, then 10s, then 100s, then 1,000s.</p> 	<p>Use the formal written method for short multiplication.</p> <p><b>M7a: Column Multiplication</b></p> $\begin{array}{r} 3647 \\ \times \quad 4 \\ \hline 14588 \end{array}$																																						
<p>Grid method (prerequisite for long multiplication).</p> <p>Y5 – Spring 1</p> <p>Some children may only be ready for this method and do not have the confidence or understanding to move onto the formal written method for long multiplication.</p>	<p>Partition one number into 10s and 1s, then add the parts.</p> <p><math>23 \times 15 = ?</math></p>   <p>There are 345 bottles of milk in total.</p> $23 \times 15 = 345$ <table border="1" data-bbox="837 1257 936 1407"> <thead> <tr> <th></th> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td></td> <td>1</td> <td>5</td> <td>0</td> </tr> <tr> <td></td> <td>1</td> <td>5</td> <td>0</td> </tr> <tr> <td>+</td> <td></td> <td>4</td> <td>5</td> </tr> <tr> <td></td> <td>3</td> <td>4</td> <td>5</td> </tr> </tbody> </table>		H	T	O		1	5	0		1	5	0	+		4	5		3	4	5	<p>Use images of place value equipment alongside grid method.</p>  <table border="1" data-bbox="1339 1043 1532 1241"> <thead> <tr> <th>x</th> <th>40</th> <th>4</th> </tr> </thead> <tbody> <tr> <th>30</th> <td>1,200</td> <td>120</td> </tr> <tr> <th>2</th> <td>80</td> <td>8</td> </tr> </tbody> </table>	x	40	4	30	1,200	120	2	80	8	<p>Use grid method.</p> <p><b>M8: Grid Method</b> Long Multiplication</p> <table border="1" data-bbox="1585 1091 1890 1273"> <thead> <tr> <th>x</th> <th>40</th> <th>3</th> </tr> </thead> <tbody> <tr> <th>60</th> <td>2400</td> <td>180</td> </tr> <tr> <th>5</th> <td>200</td> <td>15</td> </tr> </tbody> </table> $\begin{array}{r} 2400 \\ 200 \\ 180 \\ + 15 \\ \hline 2795 \end{array}$ <p><b>43 x 65 = 2795</b></p>	x	40	3	60	2400	180	5	200	15
	H	T	O																																						
	1	5	0																																						
	1	5	0																																						
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2	80	8																																							
x	40	3																																							
60	2400	180																																							
5	200	15																																							

Long multiplication.

Y5 – Spring 1  
Y6 – Autumn 2

Use manipulatives and place value equipment where appropriate.

Use drawings and images where appropriate. Teach alongside grid method to support understanding.



Use column multiplication, ensuring understanding of place value at each stage.

### M9: Long Multiplication Column

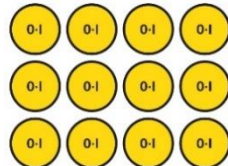
$$\begin{array}{r}
 43 \\
 \times 65 \\
 \hline
 215 \quad (5 \times 43) \\
 + 2580 \quad (60 \times 43) \\
 \hline
 2795
 \end{array}$$

Progress to include examples that require multiple exchanges as understanding, confidence and fluency build.

Year 6 – multiplying decimals.

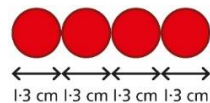
Spring 1

Explore decimal multiplications using place value equipment and in the context of measures.



3 groups of 4 tenths is 12 tenths.

4 groups of 3 tenths is 12 tenths.



$$4 \times 1 \text{ cm} = 4 \text{ cm}$$

$$4 \times 0.3 \text{ cm} = 1.2 \text{ cm}$$

$$4 \times 1.3 = 4 + 1.2 = 5.2 \text{ cm}$$

Represent calculations on a place value grid.

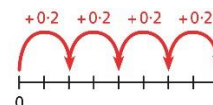
$$3 \times 3 = 9$$

$$3 \times 0.3 = 0.9$$

T	O	•	Tth

Understand the link between multiplying decimals and repeated addition.

T	O	•	Tth



Move onto column method.

### M9e: Column Multiplication

$$\begin{array}{r}
 7.38 \\
 \times 6 \\
 \hline
 44.28
 \end{array}$$

Mental methods.

Y5 – Spring 1

Y6 – Autumn 2

Throughout Years 5 and 6, emphasis must be placed on children choosing the most efficient method/strategy. They should not rely wholly on written methods. These should be taught explicitly to the children and compared alongside the written methods to show which is the most efficient. Some mental strategies include:

Round and Adjust

**MM4a: Round & Adjust**

$$198 \times 4 = 792$$

$$(200 \times 4) - (2 \times 4)$$

$$800 - 8 = 792$$

**MM4c: Round & Adjust**

$$£5.99 \times 6 = £35.94$$

$$(£6 \times 6) - (1p \times 6)$$

$$£36 - 6p = £35.94$$

Doubling and Halving

**MM6: Doubling Table Facts**

$$16 \times 7 = 112$$

(8 x 2)

$$8 \times 7 = 56$$

↓ x 2

$$16 \times 7 = 112$$

**MM7a: Doubling Up**

$$36 \times 8 = 288$$

$$\text{Double } 36 = 72 \quad (36 \times 2)$$
$$\text{Double } 72 = 144 \quad (36 \times 4)$$
$$\text{Double } 144 = 288 \quad (36 \times 8)$$

**MM9: Doubling & Halving**

$$45 \times 14$$

$$90 \times 7 = 630$$



Using Known Multiplication Facts

**MM8: Mult by  $\frac{10, 100}{1, 1000}$  then Halve**    **MM8a: Mult by  $\frac{10, 100}{1, 1000}$  then Halve**

$$86 \times 5 = 430$$

$$86 \times 10 = 860$$

$$860 \div 2 = 430$$

$$56 \times 25 = 1400$$

$$56 \times 100 = 5600$$

$$5600 \div 2 = 2800$$

$$2800 \div 2 = 1400$$

Factorising

**MM10: Factorising**

$$32 \times 15 = 480$$

$$(32 \times 5 \times 3)$$

$$160 \times 3 = 480$$

**MM10a: Factorising**

$$52 \times 24 = 1248$$

$$(52 \times 4 \times 6)$$

$$208 \times 6 = 1248$$

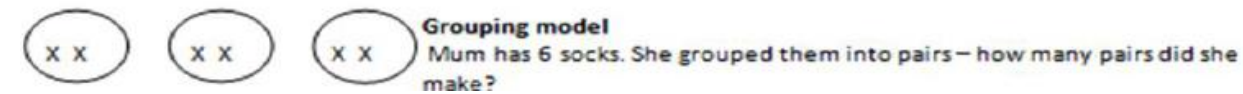
# Division – EYFS

The ELG states that children must solve problems involving doubling, halving and sharing.

Children need to see and hear different representations of both grouping and sharing.

Division can be introduced through the concept of halving.

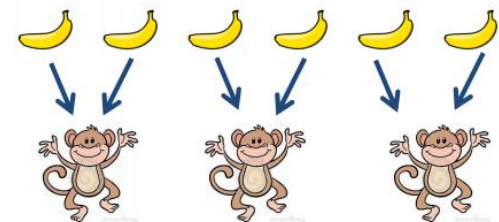
Children begin with mostly pictorial representations linked to real life contexts.



Children have a go at verbally recording the calculation and using number cards to represent it in abstract form.

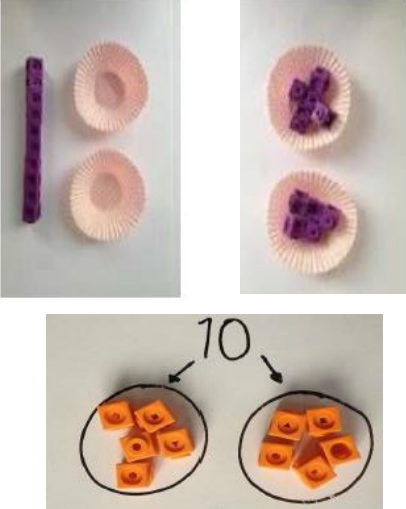
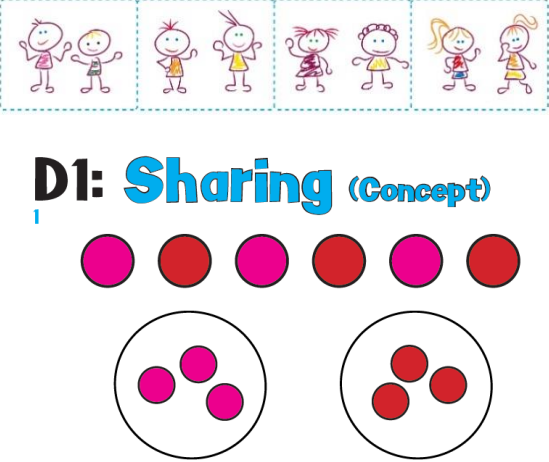


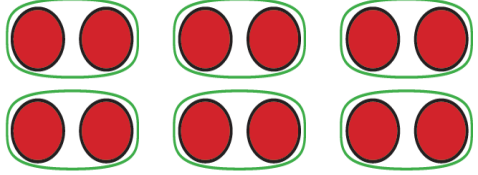
Children are encouraged to draw pictures and represent their mathematical thinking through various representations

Share the bananas fairly between the 3 monkeys. How many do they have each?


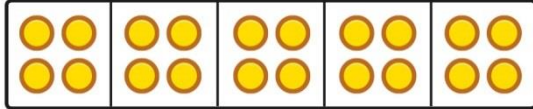
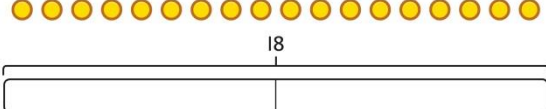





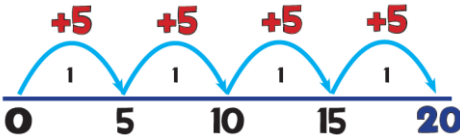





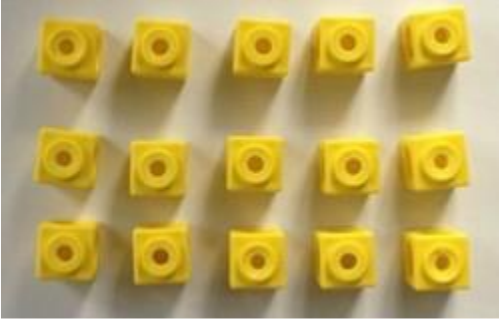
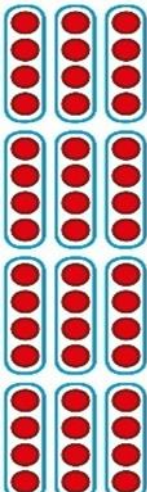
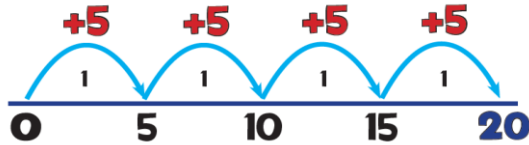
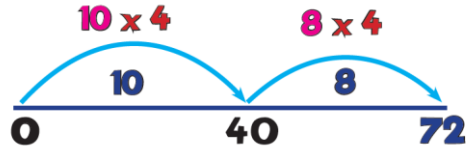
# Division – Year 1

Objective/strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p> <p>Summer 1</p>	<p>Share a set of objects into equal parts and work out how many are in each part.</p>  <p><i>I have 10 cubes, can you share them equally in 2 groups?</i></p>	<p>Sketch or draw to represent sharing into equal parts. This may be related to fractions.</p>  <p><b>D1: Sharing (Concept)</b></p> <p><i>"If I share 6 into 2 equal amounts, how many in each group?" Answer: 3</i></p>	<p>10 shared into 2 equal groups gives 5 in each group.</p> <p>12 shared between 3 is 4</p>
<p>Division as grouping.</p> <p>Summer 1</p>	<p>Learn to make equal groups from a whole and find how many equal groups of a certain size can be made.</p> <p>Sort a whole set people and objects into equal groups.</p>  <p><i>There are 10 children altogether. There are 2 in each group. There are 5 groups.</i></p>	<p>Represent a whole and work out how many equal groups.</p> <p><b>D2: Grouping (Concept)</b></p>  <p><i>"How many groups of 2 can I make out of 6?" Answer: 3</i></p>	<p><b>D4: Division as Grouping</b></p> <p><math>12 \div 2 = 6</math></p> <p><i>"How many groups of 2 can I make out of 12?" Answer: 6</i></p> 

# Division – Year 2

Objective/strategy	Concrete	Pictorial	Abstract
<p>Sharing equally.</p> <p>Autumn 4 Spring 1</p>	<p>Start with a whole and share into equal parts, one at a time.</p>  <p><i>12 shared equally between 2. They get 6 each.</i></p>	<p>Represent the objects shared into equal parts using a bar model.</p>  <p><i>20 shared into 5 equal parts. There are 4 in each part.</i></p>	<p>Use a bar model to support understanding of the division.</p>  <p><math>18 \div 2 = 9</math></p>
<p>Grouping equally.</p> <p>Autumn 4 Spring 1</p>	<p>Understand how to make equal groups from a whole.</p>  <p><i>8 divided into 4 equal groups. There are 2 in each group.</i></p>	<p>Understand the relationship between grouping and the division statements.</p> <p><math>12 \div 3 = 4</math></p>  <p><math>12 \div 4 = 3</math></p>  <p><math>12 \div 6 = 2</math></p>  <p><math>12 \div 2 = 6</math></p> 	<p>Understand how to relate division by grouping to repeated subtraction.</p> <p><b>D5: Grouping on a Number Line</b></p>  <p><b><math>20 \div 5 = 4</math></b></p> <p><small>"How many 5s in 20?" Answer: 4</small></p> <p>12 divided into groups of 3. <math>12 \div 3 = 4</math></p> <p>There are 4 groups.</p>

# Division – Year 3

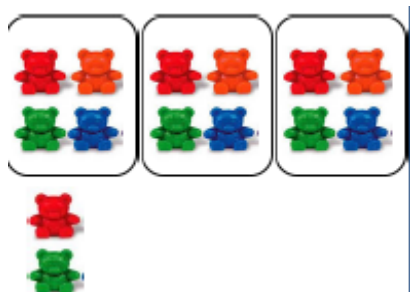
Objective/strategy	Concrete	Pictorial	Abstract
<p>Grouping using times-tables knowledge to divide.</p> <p>Spring 1</p>	<p>Use knowledge of known times-tables to calculate divisions.</p>  <p>24 divided into groups of 8. There are 3 groups of 8.</p>  <p>Create arrays to make the link clear and sort into groups.</p>	<p>Use knowledge of known times-tables to calculate divisions. Create arrays and split them into groups.</p>  <p><math>48 \div 4 = 12</math></p>	<p>Build on work in Year 2 by grouping using a number line.</p> <p><b>D5: Grouping on a Number Line</b> 2</p>  <p><b><math>20 \div 5 = 4</math></b>      "How many 5s in 20?" Answer: 4</p> <p>Children who are more confident with their multiplication should move to chunking on a number line.</p> <p><b>D7: Chunking Jump</b> 3</p>  <p><b><math>72 \div 4 = 18</math></b>      "How many 4s in 72?" Answer: 18</p>

Understanding remainders

Spring 1

$$14 \div 3 =$$

Divide objects between groups and see how much is left over:



Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.



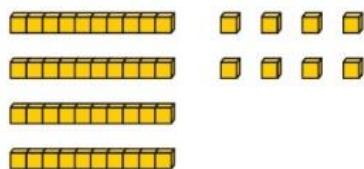
There are 13 sticks in total.

There are 3 groups of 4, with 1 remainder.

Use part-part whole models when dividing 2-digits by 1-digit.

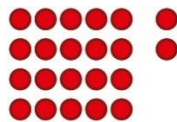
Spring 1

Children explore dividing 2-digit numbers by using place value equipment.



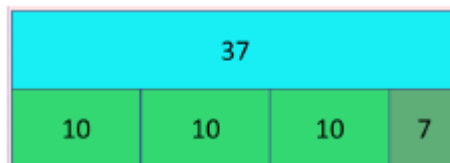
$$48 \div 2 = ?$$

Use images to explain remainders.



$$22 \div 5 = 4 \text{ remainder } 2$$

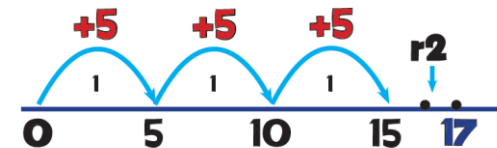
Use bar models to show remainders.



Understand that the remainder is what cannot be shared equally from a set. Chunk on a number line.

### D5a: Grouping on a Number Line

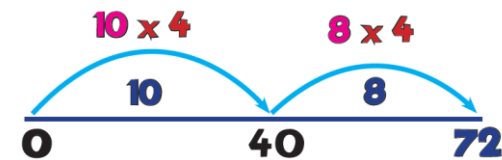
Remainders



"How many 5s in 17?"  
Answer: 3 remainder 2

$$17 \div 5 = 3r2$$

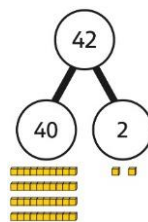
### D7: Chunking Jump



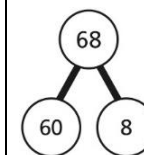
"How many 4s in 72?"  
Answer: 18

$$72 \div 4 = 18$$

Children explore which partitions support particular divisions.



Children partition a number into 10s and 1s to divide where appropriate.



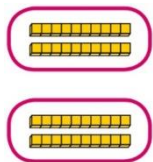
$$60 \div 2 = 30$$

$$8 \div 2 = 4$$

$$30 + 4 = 34$$

$$68 \div 2 = 34$$

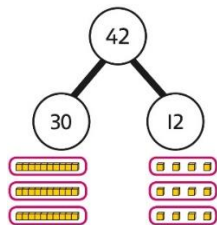
First divide the 10s.



Then divide the 1s.



I need to partition 42 differently to divide by 3.



$$42 = 30 + 12$$

$$42 \div 3 = 14$$

Children partition flexibly to divide where appropriate.

$$42 \div 3 = ?$$
$$42 = 40 + 2$$


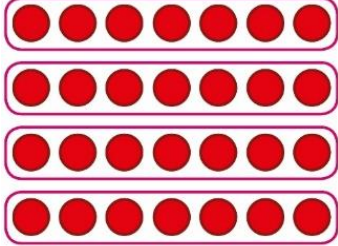
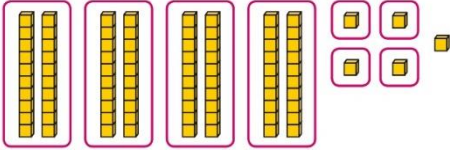
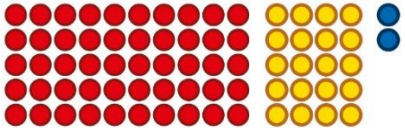
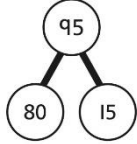
I need to partition 42 differently to divide by 3.

$$42 = 30 + 12$$

$$30 \div 3 = 10$$
$$12 \div 3 = 4$$

$$10 + 4 = 14$$
$$42 \div 3 = 14$$

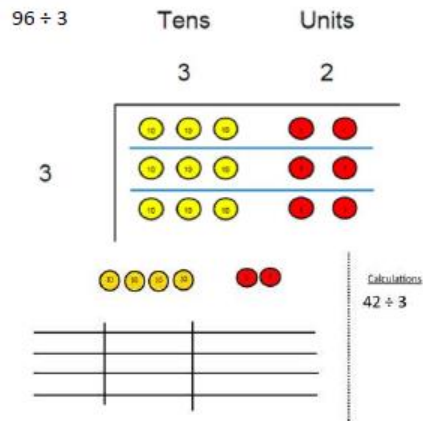
# Division – Year 4

Objective/strategy	Concrete	Pictorial	Abstract
<p>Understanding the relationship between multiplication and division, including times-tables (using arrays)</p> <p>Autumn 4 Spring 1</p>	<p>Use objects to explore families of multiplication and division facts.</p>  <p><math>4 \times 6 = 24</math> 24 is 6 groups of 4. 24 is 4 groups of 6.</p> <p>24 divided by 6 is 4. 24 divided by 4 is 6.</p>	<p>Represent divisions using an array.</p>  <p><math>28 \div 7 = 4</math></p>	<p>Understand families of related multiplication and division facts.</p> <p><i>I know that <math>5 \times 7 = 35</math></i></p> <p><i>so I know all these facts:</i></p> <p><math>5 \times 7 = 35</math>  <math>7 \times 5 = 35</math>  <math>35 = 5 \times 7</math>  <math>35 = 7 \times 5</math>  <math>35 \div 5 = 7</math>  <math>35 \div 7 = 5</math>  <math>7 = 35 \div 5</math>  <math>5 = 35 \div 7</math></p>
<p>Understanding remainders.</p> <p>Spring 1</p>	<p>Use place value equipment to find remainders.</p> <p><i>85 shared into 4 equal groups</i></p> <p><i>There are 24, and 1 that cannot be shared.</i></p> 	<p>Represent the remainder as the part that cannot be shared equally.</p>  <p><math>72 \div 5 = 14</math> remainder 2</p>	<p>Understand how partitioning can reveal remainders of divisions.</p>  <p><math>80 \div 4 = 20</math>  <math>12 \div 4 = 3</math></p> <p><math>95 \div 4 = 23</math> remainder 3</p>

Dividing 2- and 3-digit numbers by 1-digit numbers (introducing formal method for division).

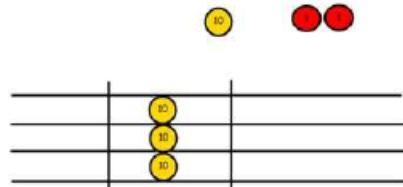
Spring 1

Use place value counters to divide using the bus stop method alongside.

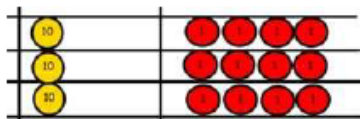


$$42 \div 3 =$$

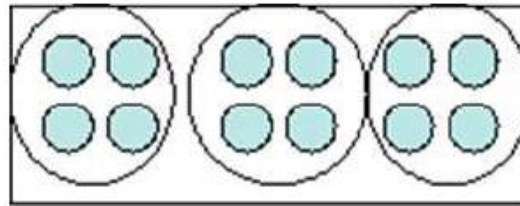
Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.



We exchange this ten for ten ones and then share the ones equally among the groups. We look how much in 1 group so the answer is 14.



Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions with no remainders.

## D10: Short Division

$$136 \div 4 = 34$$

$$\begin{array}{r} 34 \\ 4 \overline{) 136} \end{array}$$

Introduce remainders when children are secure with the method.

## D10c: Short Division

$$394 \div 6 = 65r4$$

$$\begin{array}{r} 65r4 \\ 6 \overline{) 394} \end{array}$$



# Division – Year 5

Objective/strategy y	Concrete	Pictorial	Abstract												
<p>Dividing 3- and 4-digit numbers by 1-digit numbers using formal written method (Year 6 included).</p> <p>Spring 1</p>	<p>Explore grouping using place value equipment.</p> <p><math>268 \div 2 = ?</math></p> <p>There is 1 group of 2 hundreds. There are 3 groups of 2 tens. There are 4 groups of 2 ones.</p> <p><math>264 \div 2 = 134</math></p>	<p>Use place value equipment on a place value grid alongside short division. The model uses grouping. A sharing model can also be used, although the model would need adapting.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 4 \overline{) 48} \\ \underline{4} \phantom{0} \\ 0 \phantom{0} \end{array}</math> </div> <div style="border: 1px solid black; padding: 5px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 50px; height: 30px;">T</td> <td style="width: 50px; height: 30px;">O</td> </tr> <tr> <td style="text-align: left;">(10) (10) (10)</td> <td style="text-align: left;">(1) (1) (1) (1) (1) (1)</td> </tr> </table> </div> </div> <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 1 \phantom{0} \\ 4 \overline{) 48} \\ \underline{4} \phantom{0} \\ 0 \phantom{0} \end{array}</math> </div> <div style="border: 1px solid black; padding: 5px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 50px; height: 30px;">T</td> <td style="width: 50px; height: 30px;">O</td> </tr> <tr> <td style="text-align: left;">(10) (10) (10)</td> <td style="text-align: left;">(1) (1) (1) (1) (1) (1)</td> </tr> </table> </div> </div> <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="margin-right: 10px;"> <math display="block">\begin{array}{r} 1 \phantom{0} \phantom{0} \\ 4 \overline{) 48} \\ \underline{4} \phantom{0} \\ 0 \phantom{0} \end{array}</math> </div> <div style="border: 1px solid black; padding: 5px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="width: 50px; height: 30px;">T</td> <td style="width: 50px; height: 30px;">O</td> </tr> <tr> <td style="text-align: left;">(10) (10) (10)</td> <td style="text-align: left;">(1) (1) (1) (1) (1) (1)</td> </tr> </table> </div> </div> <p>Lay out the problem as a short division.</p> <p><i>There is 1 group of 4 in 4 tens. There are 2 groups of 4 in 8 ones.</i></p> <p>Work with divisions that require exchange.</p>	T	O	(10) (10) (10)	(1) (1) (1) (1) (1) (1)	T	O	(10) (10) (10)	(1) (1) (1) (1) (1) (1)	T	O	(10) (10) (10)	(1) (1) (1) (1) (1) (1)	<p>Use short division for up to 4-digit numbers divided by a single digit.</p> <p><b>D10e: Short Division</b> 5</p> <p style="text-align: center;"><b>5978 ÷ 7 = 854</b></p> <div style="text-align: center; margin-top: 20px;"> </div> <p>Use multiplication to check.</p>
T	O														
(10) (10) (10)	(1) (1) (1) (1) (1) (1)														
T	O														
(10) (10) (10)	(1) (1) (1) (1) (1) (1)														
T	O														
(10) (10) (10)	(1) (1) (1) (1) (1) (1)														

4  $\overline{) 92}$

4  $\overline{) 92}$

4  $\overline{) 92}$

4  $\overline{) 92}$

First, lay out the problem.

How many groups of 4 go into 9 tens?  
2 groups of 4 tens with 1 ten left over.

Exchange the 1 ten left over for 10 ones.  
We now have 12 ones.

How many groups of 4 go into 12 ones?  
3 groups of 4 ones.

Understanding remainders.

Spring 1

Understand remainders using concrete versions of a problem.

80 cakes divided into trays of 6.



80 cakes in total. They make 13 groups of 6, with 2 remaining.

Use short division and understand remainders as the last remaining 1s.

6  $\overline{) 80}$

6  $\overline{) 80}$

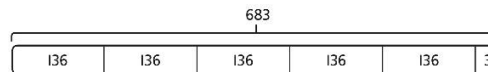
6  $\overline{) 80}$

Lay out the problem as short division.

How many groups of 6 go into 8 tens?  
There is 1 group of 6 tens.  
There are 2 tens remaining.

How many groups of 6 go into 20 ones?  
There are 3 groups of 6 ones.  
There are 2 ones remaining.

In problem solving contexts, represent divisions including remainders with a bar model.



$$683 = 136 \times 5 + 3$$

$$683 \div 5 = 136 \text{ r } 3$$

Represent the remainders depending on the context of a problem e.g. as a fraction, rounding up, rounding down or as a decimal.

**D10f: Short Division**  
Different Remainders

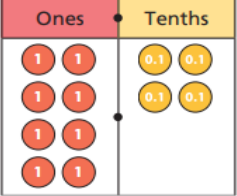
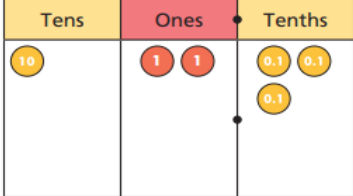
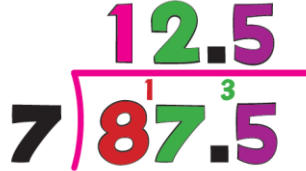
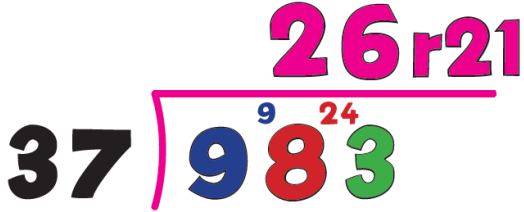
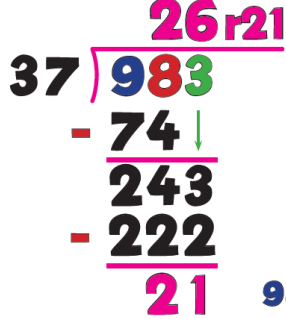
$$5 \overline{) 846.0} = 169.2$$

$$846 \div 5$$

$$5 \overline{) 846} = 169 \text{ r } 1$$

$$5 \overline{) 846} = 169 \frac{1}{5}$$

# Division – Year 6

Objective/strategy	Concrete	Pictorial	Abstract
<p>Using formal written method to divide decimal numbers by integers.</p> <p>Spring 1</p>	<p><math>8.4 \div 4 = \square</math></p>  <p><math>12.3 \div 3 = \square</math></p> 	<p>Use place value equipment to support grouping. Pupils can use the equipment or draw it out.</p>	<p>Use the formal written method for dividing decimals by integers.</p> <p><b>D10i: Short Division</b></p> <p><math>87.5 \div 7 = 12.5</math></p> 
<p>Long division.</p> <p>Autumn 2</p>	<p><b>D12: Long Division</b></p> <p><small>Short Division Method</small></p> 	<p><b>D14: Long Division</b></p> <p><small>Traditional Method</small></p>  <p><math>983 \div 37 = 26r21</math></p>	